CFD and Turbulence: Method in the Madness

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Abstract

They say in life that it's the little things that all add up to make the difference, and in many cases this is absolutely true. With a phenomenum such as turbulence spanning a broad spectrum of scales, the addage holds. This article looks at the physics of turbulence, the problems in its computational resolution, and the widely-used averaging approach used to mitigate these problems.

1 Turbulence: The Long and Short of It

When it comes to the majority of flows observed in the real-world, the transition from well-behaved laminar fluid flows to the rather more chaotic turbulent flow is a seeming inevitability. As the policing force of viscosity within the flow field gives way to the fluid inertia, transition begins to occur and the formation of turbulence eddies is seen. *Reynolds* was the first to observe this in his influential paper on fluid dynamics which ultimately lead him to propose the concept of a parameter defining the ratio of inertial to viscous forces; we of course now know this as the Reynolds number

$$Re = \frac{\rho \mathbf{u} \ell}{\mu} \tag{1}$$

As a rule of thumb, turbulent flows are typically seen at a Reynolds number in excess of 10^6 for a natural flat plate. Rough or dimpled body surfaces may also cause the onset of transition (ask any golf player).

In physical terms, turbulence manifests itself as a series of eddies: pockets of swirling fluid with rotational velocity components leading to mixing and subsequent momentum transfer within the flow. The initial, larger eddies in the flow field experience 'stretching'; this process leads to their breakdown into smaller eddies through to the smallest Kolmogorov microscales. It is these microscales which deliver the major obstacle to direct simulation of every eddy within the entire flow domain.

In the spirit of placing hard numbers on the issue of turbulent scales, consider the case where dissipation of kinetic energy occurs through viscous forces. For this to take place, inertial and viscous effects must be of a similar magnitude such that $Re \to 1$. Taking the flow medium as air at standard atmospheric conditions, this will lead to variations in velocity across a distance η in the order of 10^{-1}mm. This leads to velocity fluctuations occuring at a rate of $\eta/\overline{u} = 10^{-5}$ seconds for a mean flow velocity of $\overline{u} = 10$m/s. Industrial applications and many academic CFD studies will concern domains in the order of several millimetres through to many metres. In addition, many transient case studies span seconds or even minutes to allow full flow development. To guarantee fully accurate solutions, the computational flow field must be discretised in both spatial and temporal domains to scales much less than those at which appreciable variations occur.

2 RANS Modelling

Hopefully, the previous section will have given the reader an appreciation of the hurdles which prevent a fully direct numerical simulation (DNS) of the turbulent flow field being achieved for the overwhelming majority of CFD applications. However almost all flows include regions of turbulence which required resolving, or at the very least modelling.

Once again, *Reynolds* provides an answer that is still very much embraced within the CFD community to this day. The first stage in his approach was to recognise that the mean flow parameters were of the greatest interest and that fluctuating terms due to turbulence could be time-averaged across a period longer than that associated with the largest of turbulence motions

$$\overline{\phi}(\mathbf{x}) \equiv \frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \phi(\mathbf{x},t)dt \tag{2}$$

Through this time averaging approach, termed as a Reynolds decomposition, a flow parameter ϕ may be separated into its mean $\overline{\phi}$ and fluctuating ϕ' components such that

$$\phi(\mathbf{x},t) \equiv \overline{\phi}(\mathbf{x},t) + \phi'(\mathbf{x},t) \tag{3}$$

Substitution of the above into the Navier-Stokes equations, which govern fluid flow behaviour, provides the Reynolds-Averaged Navier-Stokes (RANS) system of equations. The continuity and momentum equations are given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \overline{u}_i}{\partial x_i} = 0 \tag{4}$$

$$\frac{\partial \rho \overline{u}_i}{\partial t} + \frac{\partial \rho \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial \overline{P}}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial \overline{u}_k}{\partial x_k}\right) - \overbrace{\rho \overline{u_i' u_j'}}^{\tau_{ij}}\right] + F_i \tag{5}$$

Observation of the above equations will show that many of the fluctuating terms are not present; this is a consequence of the time-averaging approach, where the average of a fluctuating term is $\overline{\phi'} = 0$ and thus is cancelled. The Reynolds stresses τ persist as a cross-product term and represent in physical terms the convective momentum transfer caused through the mixing due to turbulent eddies. An unfortunate consequence of the introduction of the Reynolds stress tensor is that there are more unknown variables than equations; the problem cannot be closed without further modelling.

3 Providing Closure

Having now removed the issue of resolving each minute detail of the turbulent flow field, the problem of mathematical closure is introduced with the formulation of Reynolds stresses. One possible method of modelling these stresses is to apply the Boussinesq eddy viscosity relationship, which states that

$$\tau_{ij} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) - \frac{2}{3}\rho k \delta_{ij} \tag{6}$$

where μ_t is the turbulent eddy viscosity. So-called eddy viscosity models (EVMs) base their calculations on turbulent parameters; primarily the turbulent kinetic energy k and its dissipation ϵ (other schemes may also elect to use the turbulent frequency).

4 Summary

The aim of this article was to briefly introduce the concept of turbulence modelling and why it is currently necessary in day-to-day CFD applications. To say that this is the tip of the iceberg is a gross understatement and a wide variety of literature is available on the subject for the interested reader.

The main issue preventing the direct resolution of turbulence is the high frequency, small scale nature of the chaotic array of eddies it creates. Given the large domains of interest in many CFD applications, such a fine resolution required to accurately capture each turbulent event becomes wholly impractical given the current state-of-the-art high performance computing (HPC) resources. Time averaging of flow parameters allows mean characteristics and behaviour to be captured, whilst mathematical modelling of the turbulent stresses through the implication of eddy viscosity accounts for the effect of turbulent fluid mixing.

5 Further Reading

- D. Wilcox. *Turbulence Modelling for CFD.* DCW Industries, Inc., 1994.
- H.K. Versteeg and W. Malalasekera. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method.* Pearson Education Limited, 2nd edition, 2007.