To Find

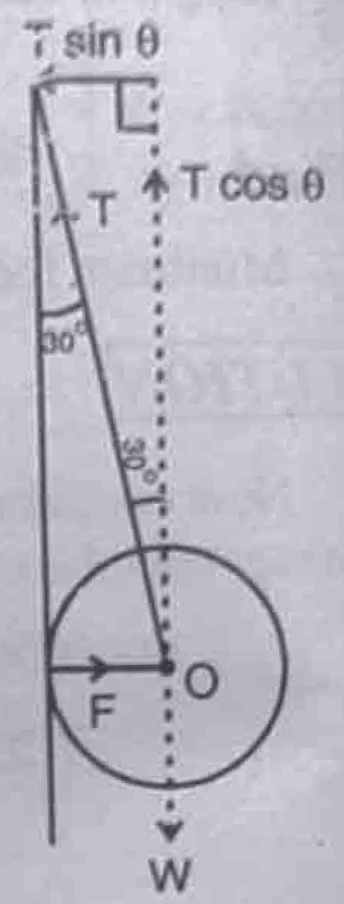
(a) Tension in the string $= T = ?$

(b) Force exerted by the wall $= F = ?$

SOLUTION

(a) Let F be the force exerted on sphere by wall, T is the tension in string and W is the weight of the sphere.

By using 1[st] condition of equilibrium



$\Sigma F_y = 0$ and $\Sigma F_x = 0$

So $F - T\sin\theta = 0$

$F = T\sin\theta$

and $T\cos\theta - W = 0$

$T\cos\theta = W$

$$T = \frac{W}{\cos\theta}$$

$$= \frac{10.0}{\cos 30} = \frac{10.0}{0.866}$$

$T = 11.55\text{ N}$

and $F = T\sin\theta$

$= 11.55 \sin 30°$

$F = 5.8\text{ N}$

Result

(a) Tension in the string $= T = 11.55\text{ N}$

(b) Force exerted by the wall $= F = 5.8\text{ N}$

Chapter 3

MOTION AND FORCE

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

★ Understand displacement from its definition and illustration.
★ Understand velocity, average velocity and instantaneous velocity.
★ Understand acceleration, average acceleration & instantaneous acceleration.
★ Understand the significance of area under velocity-time graph.
★ Recall Newton's Laws of motion.
★ Describe Newton's second law of motion as rate of change of momentum.
★ Define impulse as a product of impulsive force and time.
★ Describe law of conservation of momentum.
★ Describe the force produced due to flow of water.
★ Understand the process of rocket propulsion (simple treatment).
★ Understand projectile motion in a non-resistive medium.
★ Derive time of flight, maximum height and horizontal range of projectile motion.

Q.1 Define motion and rest.

Ans. MOTION

If a body is changing its position with respect to its surroundings then the body is said to be in motion.

REST

If a body is not changing its position with respect to some observer then the body is said to be at rest.

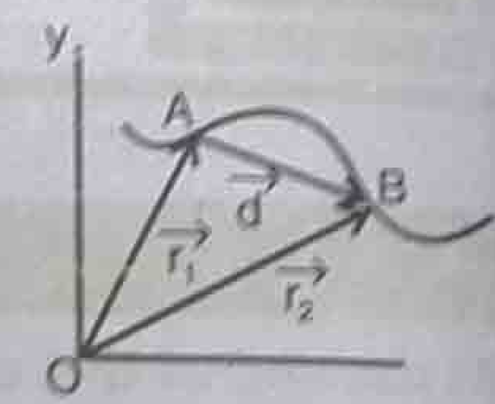
Q.2 Define displacement and distance.

Ans. DISPLACEMENT

The displacement is a change in the position of body from its initial position to its final position, shortest distance between two points is called displacement.

The displacement can be represented as a vector that describes how far and in what direction the body has been displaced from its initial position. The tail of displacement vector is located at the position where the displacement started and its tip is located at final position where displacement ended. If a body is moving along a curve as shown with A as its initial position and B as its final position then the displacement $\vec{d}$ of the body is represented by AB.

If $\vec{r}_1$ is position vector of A and $\vec{r}_2$ that of B then by head to tail rule



$$\vec{r}_1 + \vec{d} = \vec{r}_2$$

$$\therefore \quad \vec{d} = \vec{r}_2 - \vec{r}_1$$

It is a vector quantity and its SI unit is metre (m).

Distance

It is the separation between the two points. It is a scalar quantity and its SI unit is metre (m).

Q.3 *Define velocity and types of velocity.*

Ans. **VELOCITY**

The rate of change of displacement is known as velocity. Its direction is along the direction of displacement. So if $\vec{d}$ is the total displacement of the body in time t, then its average velocity during the interval t is defines as

$$\vec{V}_{av} = \frac{\vec{d}}{t}$$

It is a vector quantity and SI unit is m/s.

Dimensions

$$[\vec{V}] = m/s$$
$$= L/T$$
$$= [LT^{-1}]$$

Types of Velocity

There are three types of velocity:

(i) Uniform velocity (ii) Variable velocity (iii) Instantaneous velocity

(i) Uniform Velocity

If a body covers equal displacements in equal interval of times, however small may be interval the velocity is said to be uniform velocity.

(ii) Variable Velocity

If a body covers equal displacement in unequal interval of times however small may be the interval then it is said to be variable velocity. And its motion is non-uniform.

(iii) Instantaneous Velocity

Velocity of a body at any instant is called instantaneous velocity. **(OR)** The instantaneous velocity is also defined as the limiting value of $\frac{\Delta \vec{d}}{\Delta t}$ as the time interval Δt following the time Δt approaches to zero. Mathematically

$$\vec{V}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

Note: If the instantaneous velocity does not change the body is said to be moving with uniform velocity.

Q.4 *Define acceleration with its units.*

Ans. **ACCELERATION**

The time rate of change of velocity of a body is called acceleration. As velocity is a vector so any change in velocity may be due to change in its magnitude or change in its direction or both. Consider a body whose velocity $\vec{V}_1$ at any time t changes to $\vec{V}_2$ in small time interval Δt, therefore the change in velocity $\Delta\vec{V}$ is

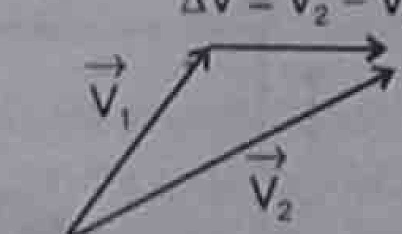
$$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$$

The average acceleration during time interval Δt is given by

$$\vec{a}_{ave} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta\vec{V}}{\Delta t}$$

If the velocity of the body is increasing then its acceleration is positive while if the velocity of the body is decreasing then its acceleration is negative. The SI unit of acceleration is m/s^2.

$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$



For Your Information

Typical Speeds	
Speed, ms^{-1}	**Motion**
300 000 000	Light, radio waves, x-rays, microwaves (in vacuum)
210 000	Earth-Sun travel around the galaxy
29 600	Earth around the Sun
1 000	Moon around the Earth
980	SR-71 reconnaissance jet
333	Sound (in air)
267	Commercial jet airliner
62	Commercial automobile (max.)
37	Falcon in a dive
29	Running cheetah
10	100-metres dash (max.)
9	Porpoise swimming
5	Flying bee
4	Human running
2	Human swimming
0.01	Walking ant

Dimensions

$$[\vec{a}] = m/s^2$$
$$= L/T^2 = [LT^{-2}]$$

Instantaneous Acceleration

Acceleration of a body at a particular instant is known as instantaneous acceleration.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t}$$

Note: *For a body moving with uniform acceleration, its average acceleration is equal to the instantaneously accelerated.*

Q.5 Explain velocity-time graph.

Ans. **VELOCITY-TIME GRAPH**

Graphs which show the variation of velocity of an object with time are called velocity-time graphs. In such graphs, the time is taken along positive x-axis because it is the independent quantity.

When velocity of car is constant

When velocity of car is constant, its velocity–time graph is a horizontal straight line as shown in Fig. (i).

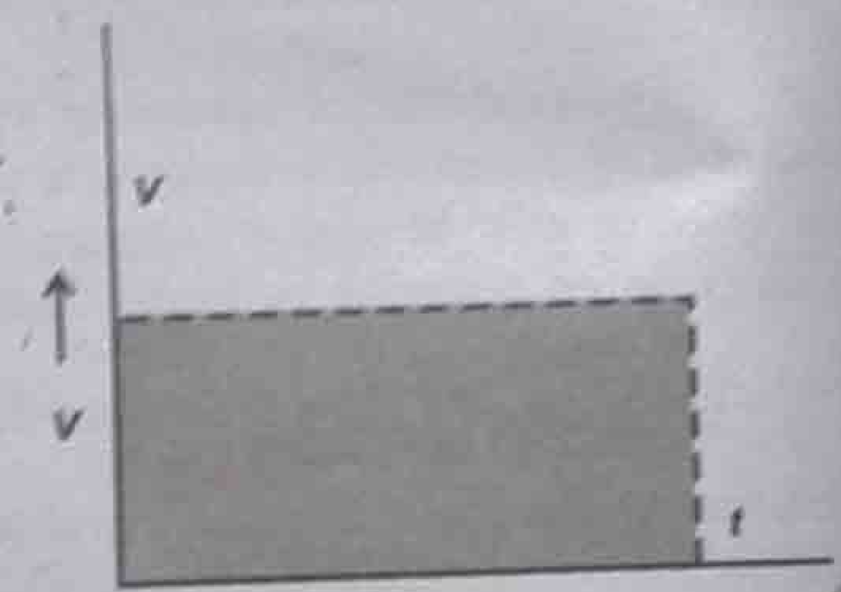


Fig. (i)

As the distance covered by the object is

$$S = vt$$

This distance moved by an object can also be found by using its velocity-time graph by calculating area under this graph. This area is shown shaded in Fig. (i).

As it is a rectangle

$\therefore$ Area under the graph $=$ Height x Width

$$H \times W = vt$$

$$A = vt$$

Hence distance covered = Area under V – t graph.

When car moves with constant acceleration

When the car moves with constant acceleration, the velocity-time graph is a straight line which rises the same height for equal intervals of time as shown in Fig. (ii).

Here the velocity of the object increased uniformly from O to V in time "t". Therefore

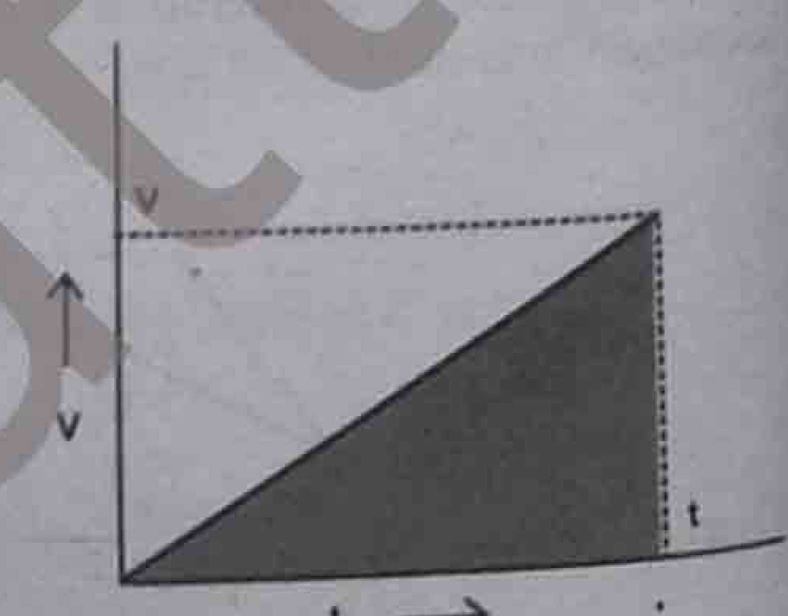


Fig. (ii)

$$\therefore \quad V_{av} = \frac{0+V}{2}$$

$$= \frac{1}{2}V$$

$$\therefore \quad S = V_{av}\, t$$

$$S = \frac{1}{2}V\,t$$

Now we calculate area under velocity-time graph which is equal to the area of the triangle shaded as shown in Fig. (ii).

$$\text{Area of } \Delta = \frac{1}{2}(\text{Base})(\text{Height})$$

$$= \frac{1}{2}(t)(v)$$

$$= \frac{1}{2}v\,t$$

Hence distance covered = area under V–t graph

"In this case average acceleration of an object can be found by finding the slope of its velocity–time graph".

Note: *The area between the velocity–time graph and the time–axis is numerically equal to the distance covered by the object.*

When the car moves with increasing acceleration

When the car moves with increasing acceleration (non–uniform velocity) the velocity-time graph is a curve as shown in Fig. (iii).

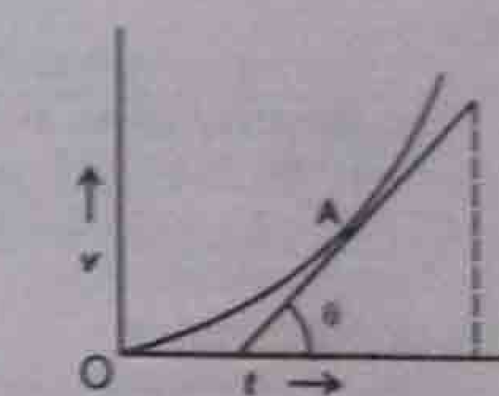


Fig. (iii)

The point A on the graph corresponds to time t. The magnitude of the Instantaneous acceleration at this instant is numerically equal to the slope of the tangent at point A on the velocity-time graph of the object as shown in Fig. (iii).

REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

Suppose an object is moving with uniform acceleration 'a' along a straight line. If initial velocity of the object is 'V_i' and final velocity 'V_f' after a time interval t. And 'S' is distance covered then we have

$$V_f = V_i + at \quad \text{...... (1)}$$

$$S = \left(\frac{V_f + V_i}{2}\right) \times t \quad \text{...... (2)}$$

$$S = V_i t + \frac{1}{2}at^2 \quad \text{...... (3)}$$

$$V_f^2 = V_i^2 + 2aS \quad \text{...... (4)}$$

These equations are useful only for linear motion with uniform acceleration.

When the object moves along the straight line, the direction of motion does not change. In this case all the vector can be manipulated like scalars. In such problems the direction of initial is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity.

In the absence of air resistance, all objects near the surface of earth, moves towards the earth with a uniform acceleration. This acceleration, is known as acceleration due to gravity. It is denoted by 'g'. Its average value near the earth surface is taken as 9.8 ms^{-2} in the down ward direction.

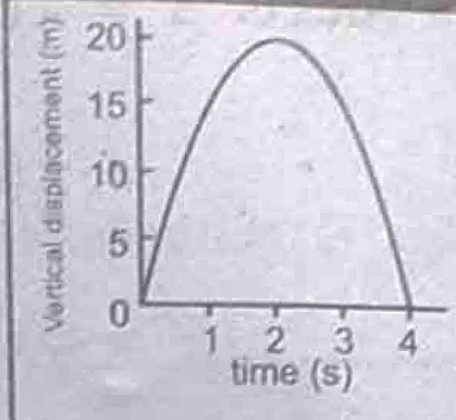
Note: *The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing 'a' by 'g'.*

Q.6 State and explain Newton's laws of motion.

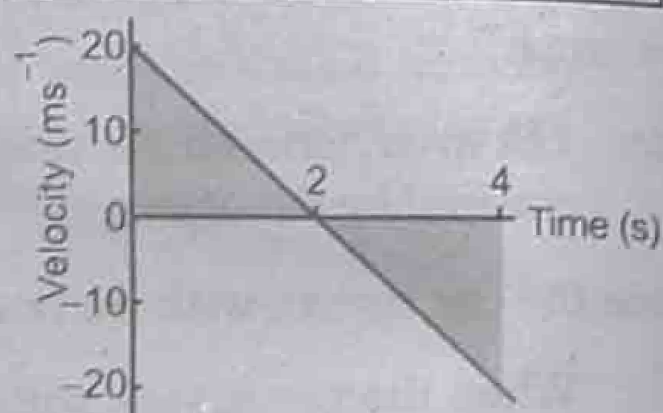
Ans. **NEWTON'S LAWS OF MOTION**

Newton's laws are empirical laws deduced from experiments. They were clearly stated for the 1st time by Sir Isaac Newton who published them in 1687 in his famous book called "Principia". Newton's laws are applicable only for speed which is negligible compared to speed of light. For very fast moving objects, such as atomic particle in an accelerator, relativistic mechanics developed by Einstein is applicable.

Do You Know?



How the displacement of a vertically thrown ball varies with time.



How the velocity of a vertically thrown ball varies with time? Velocity is upwards positive.

NEWTON'S FIRST LAW OF MOTION

A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force. This is also known as Law of "Inertia".

Inertia

The property of an object tending to maintain to the state of rest or state of uniform motion is known as object's inertia. The mass of the object is a quantitative measure of its inertia.

Frame of Reference

The space bounded by three mutually perpendicular lines is known as frame of reference. There are two types:

(i) Inertial frame of reference

The frame of reference in which Newton's laws of motions holds is known as inertial frame of reference. It is non-accelerated frame of reference.

(ii) Non-inertial frame of reference

A frame of reference in which Newton's laws of motions does not hold is known as non-inertial frame of reference. It is accelerated frame of reference.

e.g., A frame of reference stationed on earth is approximately an inertial frame of reference.

Do You Know?

At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

NEWTON'S SECOND LAW OF MOTION

A force applied on a body produces acceleration in its own direction. The acceleration produced is directly proportional with the applied force and inversely proportional with the mass of the body.

Mathematically, it is expressed as

$$\therefore \quad \vec{a} \propto \vec{F} \qquad \text{.........(i)}$$

$$\vec{a} \propto \frac{1}{m} \qquad \text{.........(ii)}$$

Combining (i) and (ii)

$$\vec{a} \propto \frac{\vec{F}}{m}$$

$$\vec{a} = k\frac{\vec{F}}{m}$$

where K = constant of proportionality

If F = 1 N, m = 1 kg

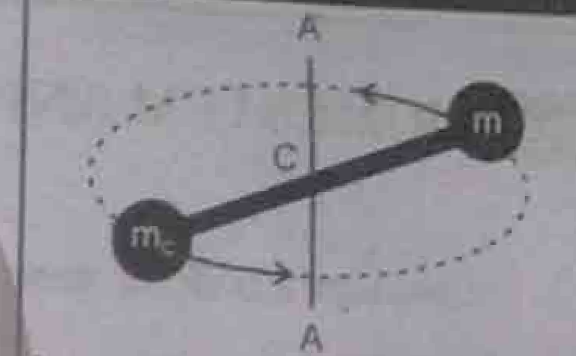
a = 1 m/s^2

then K = 1

If S.I. units are used then

$$\therefore \quad \vec{F} = m\vec{a}$$

For Your Information



A measurement of mass independent of gravity. The unknown mass m and a calibrated mass m$_c$ are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?

Ans. Force of friction.

NEWTON'S THIRD LAW OF MOTION

Action and reaction are equal but in opposite direction.

For example, whenever an interaction occurs between two objects, each object exerts the same force on the other, but in the opposite direction and for the same length of time. Each force in action-reaction pair acts only on one of the two bodies, the action and reaction forces never act on the same body.

Q.7 What is the linear momentum? Also define its units.

Ans. **MOMENTUM (LINEAR MOMENTUM)**

It is defined as the "the product of mass and velocity of the object."

It is denoted by "P".

Mathematically

$$\vec{P} = m\vec{V}$$

or The quantity of motion in a moving body is called linear momentum.

Linear momentum is a vector quantity and has the direction in direction of velocity.

The magnitude of momentum depends upon the mass of body and velocity of the body.

Unit

The SI unit of momentum is kg m/s. It is also Ns.

Dimensions

$$[P] = \text{Kg m/s} = \text{ML/T} = [\text{MLT}^{-1}]$$

Q. Show that kg m/s is equal to Ns?

Ans. Kg m/s = Ns

As L.H.S. = Kg m/s

Multiple and divide by s

$$= \text{Kg m/s} \times \frac{s}{s}$$

$$= [\text{Kg m/s}^2] \times s$$

$$= \text{Ns}$$

$$= \text{R.H.S}$$

Q.8 *How force and linear momentum are related? (OR) State Newton's second law of motion in terms of momentum.*

Ans. MOMENTUM AND NEWTON'S SECOND LAW OF MOTION

Consider a body of mass 'm' moving with an initial velocity $\vec{V}_i$. Suppose an external force $\vec{F}$ acts upon it for time 't' after which velocity becomes $\vec{V}_f$.

As,

$$\vec{V}_f = \vec{V}_i + \vec{a}\,t$$

$$\vec{a}\,t = \vec{V}_f - \vec{V}_i$$

$$\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{t} \qquad \text{.......... (1)}$$

From Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}}{m} \qquad \text{.......... (2)}$$

From equation (1) and (2)

$$\frac{\vec{F}}{m} = \frac{\vec{V}_f - \vec{V}_i}{t}$$

$$\vec{F} = \frac{m\vec{V}_f - \vec{V}_i}{t}$$

$$\vec{F} = \frac{\vec{P}_f - \vec{P}_i}{t}$$

$$\vec{F} = \frac{\Delta \vec{P}}{t}$$

Hence second law of motion in term of momentum can also be stated as "the time rate of change of momentum of a body equals the applied force."

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move outward from shore (Newton's third law).

Point to Ponder

Which will be more effective in knocking a bear down.
i. a rubber bullet or
ii. a lead bullet of the same Momentum

Ans. Rubber bullet will be more effective in knocking a bear down because its rate of change of momentum will be greater than that of lead bullet.

Q.9 *Define impulse and show that it is change in momentum.*

Ans. IMPULSE

When a very large force acts on a body for a very short interval of time the momentum of the body changes. The product of such a force and time is called the impulse. It is denoted by I and it is a vector quantity.

$$\therefore \text{Impulse} = \vec{I} = \vec{F} \times t$$

As

$$\vec{F} = \frac{m\vec{V}_f - m\vec{V}_i}{t}$$

$$\vec{F} \times t = m\vec{V}_f - m\vec{V}_i$$

$$\therefore \quad \text{Impulse} = \text{change in momentum } (\Delta\vec{P})$$

Unit: Its unit is Kg ms^{-1} or Ns.

Point to Ponder

Which hurt you in the above situations (a) or (b) and think why?

Point to Ponder

Does a moving object have impulse?

Ans. There are two possibilities:
i. If a body is moving with constant velocity then change in momentum will be zero therefore impulse will be zero but if a body moves with variable velocity then there will be change in momentum and then the moving body will have impulse.

Do You Know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Q.10 *State and explain law of conservation of linear momentum.*

Ans. LAW OF CONSERVATION OF MOMENTUM

Isolated System

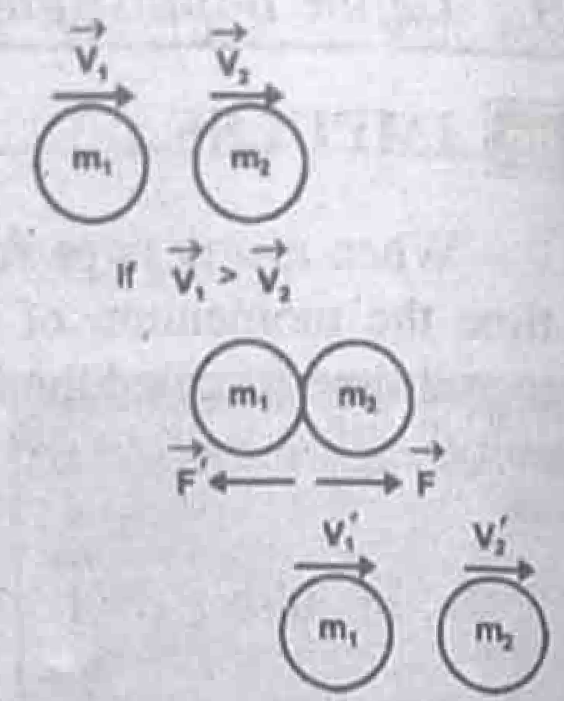
It is a system on which no external agency exerts any force. e.g., The molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion but, no external force can exert on them.

Statement

This law states that the total linear momentum of an isolated system remains constant.

Explanation

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively. Both the balls collide and after collision, the ball of mass m_1 moves with velocity $\vec{V}_1'$ and m_2 moves with velocity $\vec{V}_2'$ in the same direction as shown in figure.



To find the change in momentum we use

$$\vec{F} \times t = m\vec{V}_f - m\vec{V}_i$$

For mass m_1

$$\vec{F} \times t = m_1\vec{V}_1' - m_1\vec{V}_1 \quad \text{......... (1)}$$

Similarly for mass m_2

$$\vec{F}' \times t = m_2\vec{V}_2' - m_2\vec{V}_2 \quad \text{......... (2)}$$

Adding (1) and (2)

$$\vec{F} \times t + \vec{F}' \times t = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$(\vec{F} + \vec{F}')t = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

Since the action F is equal and opposite to the reaction force F′

i.e., $$\vec{F}' = -\vec{F}$$

$$\therefore \quad t(\vec{F} - \vec{F}) = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$0 = m_1\vec{V}_1' - m_1\vec{V}_1 + m_2\vec{V}_2' - m_2\vec{V}_2$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

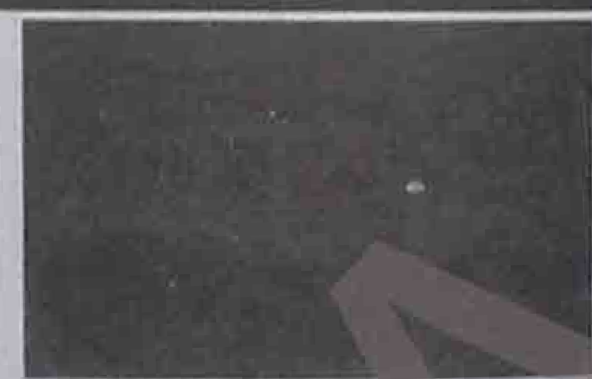
which means that total initial momentum of the system before collision is equal to the final momentum of the system after collision. Consequently the total change in momentum of the isolated two ball system is zero.

Point to Ponder

What is the effect on the speed of a fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?

Ans. The speed of fighter plane chasing another will decrease due to law of conservation of momentum. While the speed of pursued plane will increase.

Do You Wear Seat Belts?



When a moving car stops quickly, the passengers move forward towards the windshield. Seat belts change the forces of motion and prevent the passengers from moving. Thus the chance of injury is greatly reduced.

Do You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Q.11 Define elastic and inelastic collision.

Ans. **ELASTIC AND INELASTIC COLLISIONS**

Collision

When two or more object come close enough so that there is some sort of interaction between them, with or without the presence of external force, we say a collision has been taken place between the objects.

There are two types of collision:

1. Head-on collision, such a collision in which after collision balls move in same direction as they move before collision.
2. Oblique collision (direction of balls changes after collision).

Elastic Collision

In the ideal case when no K.E is lost, the collision is said to be perfectly elastic.

For example, when a hard ball is dropped on to a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor.

Inelastic Collision

A collision in which the Kinetic Energy of the system is not conserved is called Inelastic Collision.

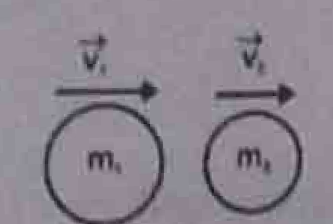
When two tennis balls collide then after collision, they will rebound with velocities less than the velocities before the impact. During this process, a portion of K.E. is lost, partly due to friction as the molecules in the ball move past one another when the balls distort and partly due to its change into heat and sound energies.

Note: Momentum and total energy are conserved in all types of collisions.

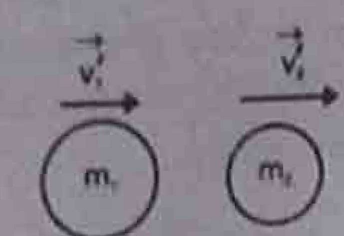
Q.12 Discuss elastic collision in one dimension and prove that speed of approach speed of released. (OR) Derive the formula of final velocities of two balls after an elastic collision in one dimensions.

Ans. **ELASTIC COLLISION IN ONE DIMENSION**

Consider two smooth, non–rotating balls of masses m_1 and m_2 moving initially with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after collision be $\vec{V}_1'$ and $\vec{V}_2'$ respectively, as shown in figure.



Before collision



After collision

Consider direction of the velocity and momentum to the right.

Since the collision is elastic therefore both momentum and K.E. are conserved.

By Appling Law of conservation of momentum

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2' \quad ....(1)$$

$$m_1\vec{V}_1 - m_1\vec{V}_1' = m_2\vec{V}_2' - m_2\vec{V}_2$$

$$m_1(\vec{V}_1 - \vec{V}_1') = m_2(\vec{V}_2' - \vec{V}_2) \quad ....(2)$$

Using law of conservation of K.E.

$$\frac{1}{2}m_1\vec{V}_1^2 + \frac{1}{2}m_2\vec{V}_2^2 = \frac{1}{2}m_1\vec{V}_1'^2 + \frac{1}{2}m_2\vec{V}_2'^2$$

$$\frac{1}{2}\left(m_1\vec{V}_1^2 + m_2\vec{V}_2^2\right) = \frac{1}{2}\left(m_1\vec{V}_1'^2 + m_2\vec{V}_2'^2\right)$$

$$m_1\vec{V}_1^2 + m_2\vec{V}_2^2 = m_1\vec{V}_1'^2 + m_2\vec{V}_2'^2$$

$$m_1\vec{V}_1^2 - m_1\vec{V}_1'^2 = m_2\vec{V}_2'^2 - m_2\vec{V}_2^2$$

$$m_1\left(\vec{V}_1^2 - \vec{V}_1'^2\right) = m_2\left(\vec{V}_2'^2 - \vec{V}_2^2\right)$$

$$m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_1') = m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2) \quad ....(3)$$

Dividing equation (3) by equation (2)

$$\frac{m_1(\vec{V}_1 - \vec{V}_1')(\vec{V}_1 + \vec{V}_1')}{m_1(\vec{V}_1 - \vec{V}_1')} = \frac{m_2(\vec{V}_2' - \vec{V}_2)(\vec{V}_2' + \vec{V}_2)}{m_2(\vec{V}_2' - \vec{V}_2)}$$

$$\vec{V}_1 + \vec{V}_1' = \vec{V}_2' + \vec{V}_2 \quad ....(4)$$

$$\vec{V}_1 - \vec{V}_2 = \vec{V}_2' - \vec{V}_1'$$

$$\vec{V}_1 - \vec{V}_2 = -(\vec{V}_1' - \vec{V}_2')$$

$$\vec{V}_{rel} = -\vec{V}_{rel}'$$

Before collision $(\vec{V}_1 - \vec{V}_2)$ is the velocity of first ball relative to second ball. Similarly $(\vec{V}_2' - \vec{V}_1')$ is the velocity of second ball relative to first ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation. i.e.,

$$\left\{\text{Magnitude of relative velocity of approach}\right\} = \left\{\text{Magnitude of relative velocity of separation}\right\}$$

Calculation of Velocity $\vec{V}_1'$ and $\vec{V}_2'$:

From equation (4)

$$\vec{V}_1 + \vec{V}_1' = \vec{V}_2' + \vec{V}_2$$

$$\vec{V}_2' = \vec{V}_1 + \vec{V}_1' - \vec{V}_2$$

Put this value in equation (1)

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\left(\vec{V}_1 + \vec{V}_1' - \vec{V}_2\right)$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_1 + m_2\vec{V}_1' - m_2\vec{V}_2$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_2\vec{V}_2 - m_2\vec{V}_1 = (m_1 + m_2)\vec{V}_1'$$

Dividing both sides by $(m_1 + m_2)$

$$\frac{(m_1 - m_2)\vec{V}_1 + 2m_2\vec{V}_2}{(m_1 + m_2)} = \frac{(m_1 + m_2)\vec{V}_1'}{(m_1 + m_2)}$$

$$\vec{V}_1' = \frac{(m_1 - m_2)\vec{V}_1}{(m_1 + m_2)} + \frac{2m_2\vec{V}_2}{(m_1 + m_2)} \quad ....(5)$$

From equation (4)

$$\vec{V}_1' = \vec{V}_2' + \vec{V}_2 - \vec{V}_1$$

Put this value in equation (1)

$$\therefore \quad m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\left(\vec{V}_2' + \vec{V}_2 - \vec{V}_1\right) + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_2' + m_1\vec{V}_2 - m_1\vec{V}_1 + m_2\vec{V}_2'$$

$$m_1\vec{V}_1 + m_2\vec{V}_2 + m_1\vec{V}_1 - m_1\vec{V}_1 = (m_1 + m_2)\vec{V}_2'$$

$$2m_1\vec{V}_1 + (m_2 - m_1)\vec{V}_2 = (m_1 + m_2)\vec{V}_2'$$

Dividing both sides by $(m_1 + m_2)$

$$\therefore \quad \vec{V}_2' = \frac{2m_1\vec{V}_1}{m_1 + m_2} + \frac{(m_2 - m_1)\vec{V}_2}{m_1 + m_2} \quad ....(6)$$

Do You Know?

If another car crashes into back of yours, the head-rest of the car seat can save you from serious neck injury. It helps to accelerate your head forward with the same rate as the rest of your body.

Point to Ponder

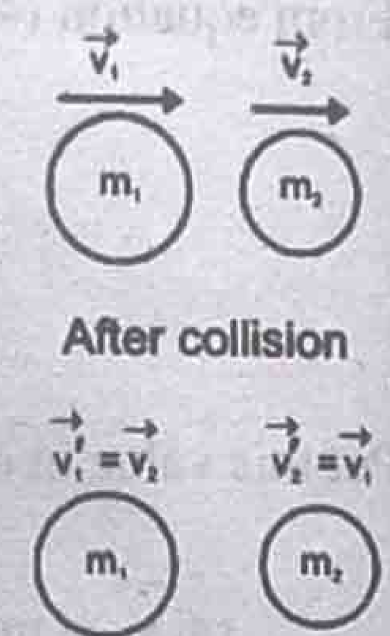
In thrill machine rides at amusement parks, there can be an acceleration of 3g or more. But without head rests, acceleration like this would not be safe. Think why not?

Q.13 *Discuss the various cases of elastic collision in dimensions.*

Ans. **SPECIAL CASES:**

Case-I: When $m_1 = m_2 = m$

Putting this in equation (5) and equation (6)



$$\therefore \quad \vec{V}_1' = \frac{(m-m)\vec{V}_1}{m+m} + \frac{2m_2\vec{V}_2}{m+m}$$

$$= 0 + \frac{2m\vec{V}_2}{2m} = \frac{2m\vec{V}_2}{2m}$$

$$\vec{V}_1' = \vec{V}_2$$

Now
$$\vec{V}_2' = \frac{2m\vec{V}_1}{m+m} + \frac{(m-m)\vec{V}_2}{m+m}$$

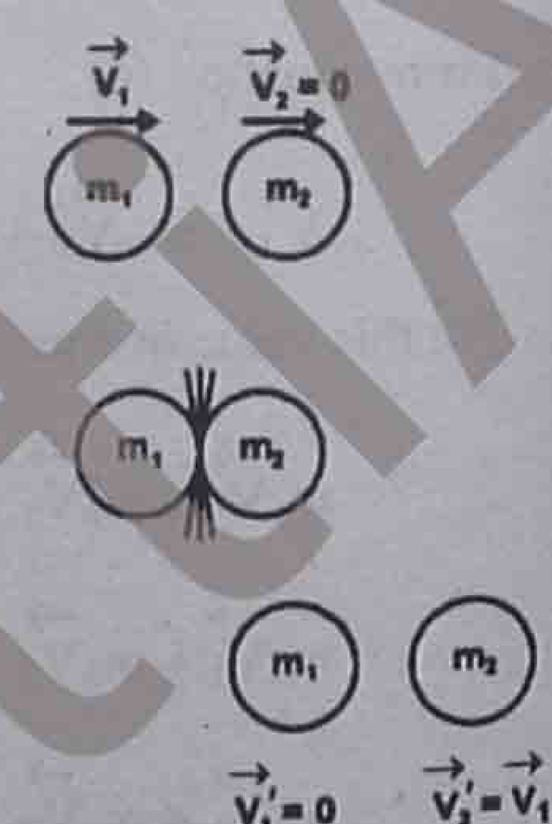
$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}_1$$

It means that when two balls of equal mass collide elastically, they simply exchange their velocities.

Case-II: When $m_1 = m_2 = m$ and $\vec{V}_2 = 0$ i.e., target ball at rest

Put this value in eq. (5) and (6)



$$\therefore \quad \vec{V}_1' = \frac{(m-m)\vec{V}_1}{m+m} + \frac{2m\,(0)}{m+m}$$

$$= 0 + 0$$

$$\vec{V}_1' = 0$$

Now,
$$\vec{V}_2' = \frac{2m\vec{V}_1}{m+m} + \frac{(m-m)\,(0)}{m+m}$$

$$= \frac{2m\vec{V}_1}{2m} + 0$$

$$\vec{V}_2' = \vec{V}$$

In this case the ball m_1 comes to rest after collision while ball m_2 that was at rest began to move with $\vec{V}_1$.

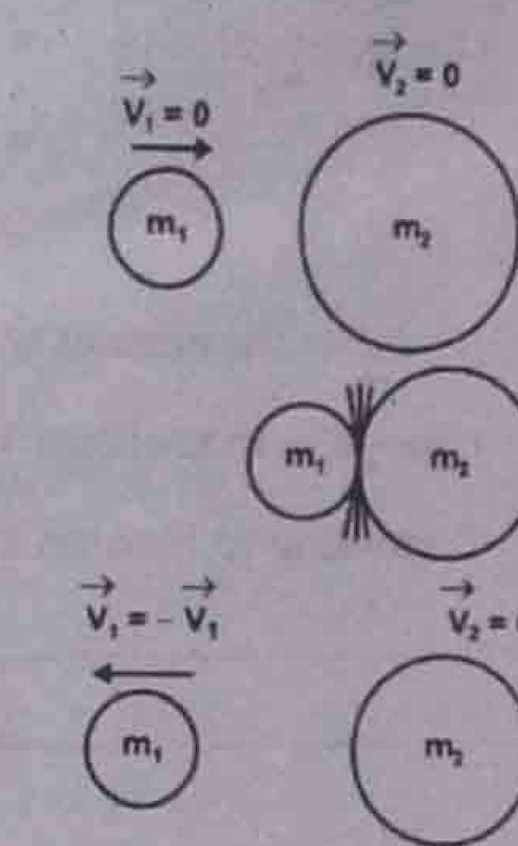
Case-III: When a light body collides with a massive body which is at rest.

i.e., $\vec{V}_2 = 0$

also $m_2 >> m_1$

i.e., $m_1 \simeq 0$

Putting this value in equation (5) and equation (6).



$$\therefore \quad \vec{V}_1' = \frac{0-m_2}{0+m_2}\vec{V}_1 + \frac{2m_2}{0+m_2}(0)$$

$$= -\frac{m_2}{m_2}\vec{V}_1$$

$$\vec{V}_1' = -\vec{V}_1$$

Also
$$\vec{V}_2' = \frac{m_2-m_1}{m_1+m_2}\vec{V}_2 + \frac{2m_1\vec{V}_1}{m_1+m_2}$$

$$= 0 + \frac{2\,(0)\,\vec{V}_1}{0+m_2}$$

$$= 0 + 0$$

$$\vec{V}_2' = 0$$

This means that m_1 will bounce back with same velocity while m_2 remains stationary.

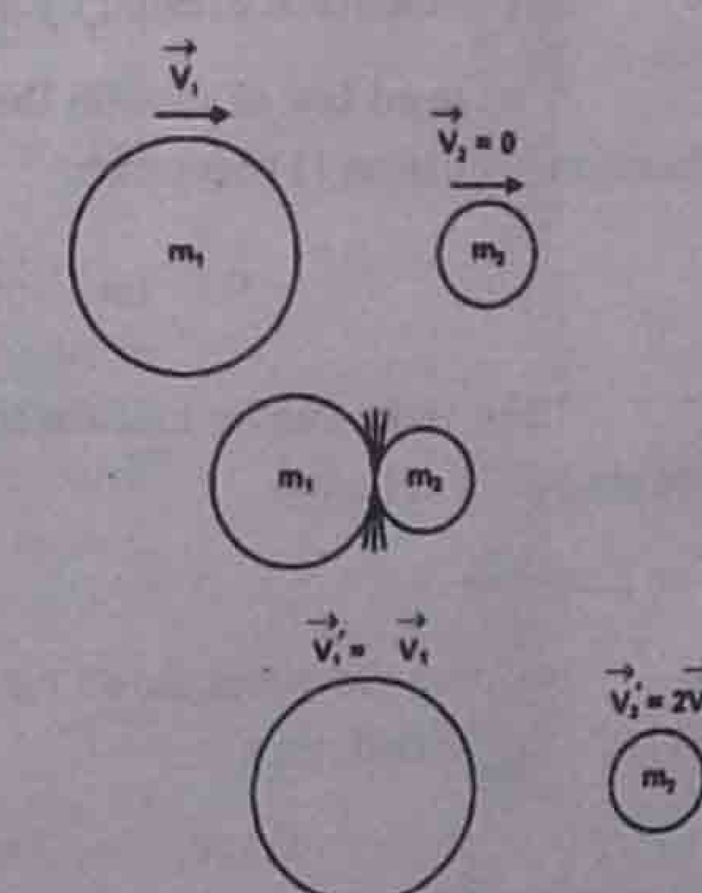
Case-IV: When a massive body collides with a lighter body at rest.

i.e., $\vec{V}_2 = 0$

As $m_1 >> m_2$

$\therefore$ $m_2 \simeq 0$

Putting this value in equation (5) and equation (6).



$$\vec{V}_1' = \frac{m_1-0}{m_1+0}\vec{V}_1 + \frac{2\,(0)\,(0)}{m_1+0}$$

$$= \frac{m_1\vec{V}_1}{m_1} + 0$$

$$\vec{V}_1' = \vec{V}_1$$

Also
$$\vec{V}_2' = \frac{0-m}{m_1-0}(0) + \frac{2m_1\vec{V}_1}{m_1+0}$$

$$= 0 + \frac{2m_1\vec{V}_1}{m_1}$$

$$\vec{V}_2' = 2\vec{V}_1$$

Hence after the collision there is practically no change in the velocity of massive body but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body.

Q.14 *Derive a relation for the force due to water flow on the wall.*

Ans. FORCE DUE TO WATER FLOW

When water from a horizontal pipe strikes a wall normally, a force is exerted on the wall. Suppose the water strikes the wall normally with velocity V and comes to rest on striking the wall, then

$$\text{Change in velocity} = \text{Final velocity} - \text{Initial velocity}$$
$$= 0 - V$$
$$= -V$$

If 'm' is the mass of water that strikes the wall, then

$$\text{Change in momentum} = m(-V) = -mV$$

According to Newton's second law, time rate of change of momentum is equal to force applied i.e.,

$$F = \frac{\text{Change in momentum}}{\text{time}}$$

or

$$F = -\frac{m\,v}{t} = -\left(\frac{m}{t}\right)v \qquad \text{.........(1)}$$

or

$$\text{Force} = -(\text{Mass per second}) \times (\text{Change in velocity})$$

This is the force exerted by the wall on water.

From third law of motion the reaction force exerted by water upon the wall is equal but opposite. Therefore, equation (1) becomes

$$F = -\frac{(-m\,v)}{t} = \frac{m\,v}{t}$$

Thus, force can be calculated from the product of the mass of water striking normally per second and change in velocity.

For Example

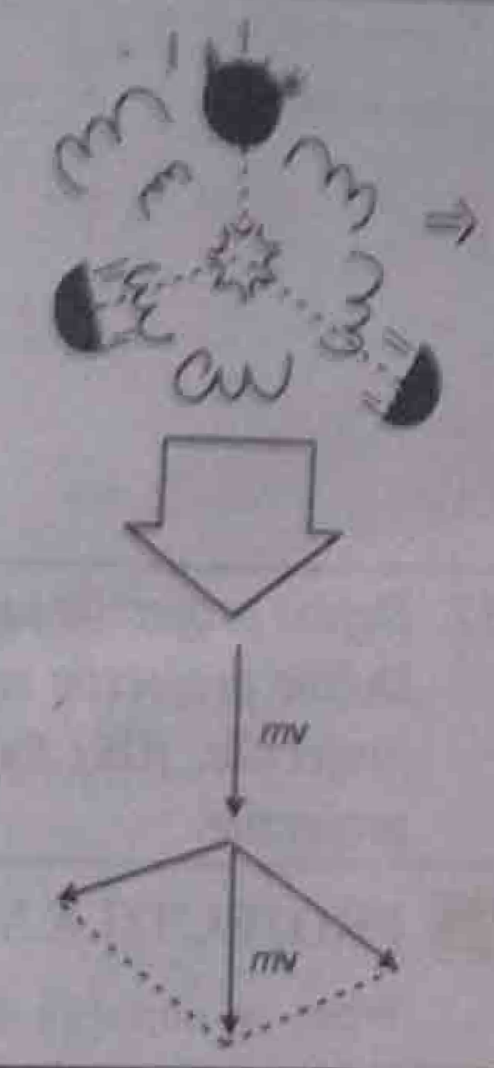
Suppose the water flows out from a pipe at 3 kgs^{-1} and its velocity changes from 5 ms^{-1} to zero on striking the ball, then,

$$\text{Force} = 3\ kgs^{-1} \times (5\ ms^{-1} - 0) = 15\ kgms^{-2} = 15\ N$$

Q.15 *Write a note on momentum and explosive forces.*

Ans. MOMENTUM AND EXPLOSIVE FORCES

There are many examples where momentum changes are produced by explosive forces within an isolated system for example, when a shell explodes in mid-air, fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell. Suppose a falling bomb explodes into pieces as shown in figure. The momenta of the bomb fragments combine by vector addition to equal to original momentum of the falling bomb.



Consider another example of bullet of mass m fired from the rifle of mass M with a velocity v. Initially, the total momentum of the bullet and rifle is zero. From the principle of conservation of linear momentum, when the bullet fired, the total momentum of bullet and rifle still remain zero, since no external force has acted on them. Thus if V′ is the velocity of the rifle then

$$\therefore \quad 0 + 0 = mV + MV'$$
$$0 = mV + MV'$$
$$MV' = -mV$$
$$V' = -\frac{mV}{M}$$

The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than bullet, it follows that the rifle moves back or rifle with only a fraction of velocity of the bullet.

Q.16 *What do you know about rocket propulsion?*

Ans. ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ingited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines. The rocket gains momentum to the momentum of the gas expelled from the engine it in opposite direction. The rocket engines continue to gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So instead of traveling at steady speed the rocket gets faster and faster sol long the engines are operating.

Rocket carries its own fuel in the form of a liquid or solid and oxygen. It can, therefore, work at great heights where little or no air is present. In order to provide enough inward thrust to overcome gravity, a typical rocket resumes about 10000 kgs^{-1} of fuel and eject the burnt at speed of over 4000 ms^{-1}. In effect, more than 10% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to take the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving to carry the space craft further up at every greater speed.

If m is the mass of the gases ejected per second with velocity relative to the rocket, the change in momentum per second of ejecting gases is mv. This equals the thrust produced in by the engine on the body of the rocket. So, the acceleration, $\vec{a}$ of the rocket is.

$$\vec{F} = \frac{m\vec{V}}{t}$$

$$\text{As} \qquad t = 1\ \text{sec}$$

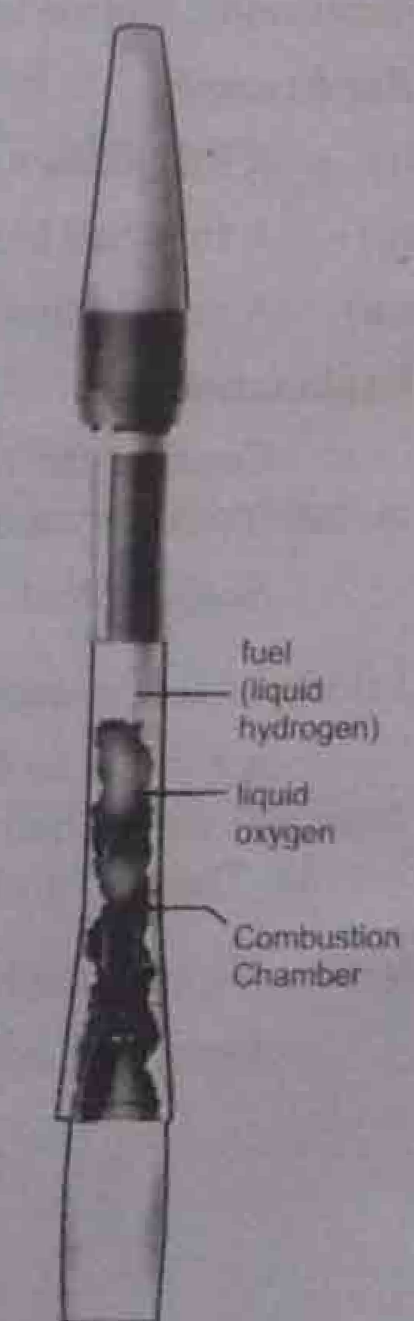


Fig. Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals. The gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

$$\vec{F} = \frac{m\vec{V}}{1}$$

$$\vec{F} = m\vec{V}$$

$$M\vec{a} = m\vec{V}$$

$$\vec{a} = \frac{m\vec{V}}{M}$$

Q.17 *Define projectile motion. Also calculate the velocity of the projectile of any instant. (OR) Define projectile motion. Also derive the formulae for the time of flight and height of projectile. (OR) Explain projectile motion. Also derive the formula for the range of projectile.*

Ans. PROJECTILE MOTION

When an object is thrown in air making a certain angle with horizontal, so that object moves under the action of gravity and moves along a curved path, is called as "projectile". Its motion is called "projectile motion". Its path is called trajectory. Its path is parabolic. **(OR)** Projectile motion is two dimensional motion under constant acceleration due to gravity.

For Example:

(i) A ball thrown by a cricketer

(ii) A foot-ball kicked by a player.

(iii) A missile fired from a launching pad.

Explanation

Consider the motion of a ball when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downwards, until it strikes something.

Suppose that the ball leaves the hand of the thrower at point A, as shown in figure. And its velocity at that instant is completely horizontal, i.e., $\vec{V}_x$.

according to Newton's 1st law of motion, there will be no acceleration in horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight, is the force of gravity. There is no horizontal force acting on it, so its horizontal velocity will remain unchanged and will be $\vec{V}_x$, until the ball hits something.

Hence, the horizontal distance is

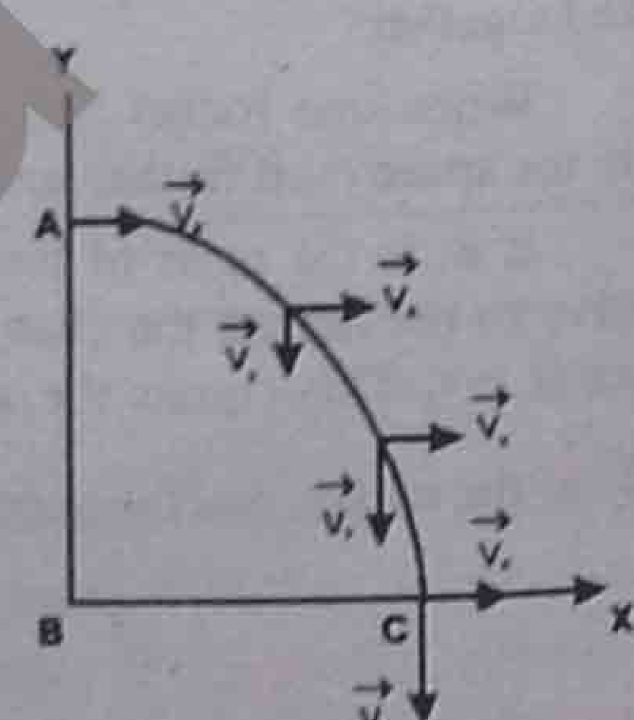
$$S = V_i t + \frac{1}{2} a t^2$$

$$x = V_{ix} t + \frac{1}{2} a_x t^2$$

$\because \quad V_x = \text{Constant}$

$\therefore \quad a_x = 0$

$\therefore \quad X = V_{ix} t$



This is the horizontal distance.

The ball will accelerate downward under the force of gravity and hence $\vec{a} = g$. This vertical motion is the same as for a freely falling body.

As, $\quad y = V_{iy} t + \frac{1}{2} a_y t^2$

$\therefore \quad V_{iy} = 0$

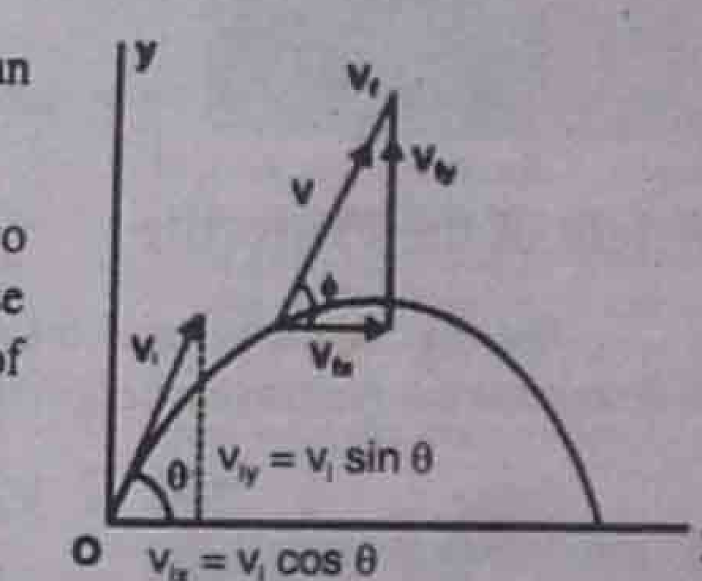
$a_y = g$

$\therefore \quad y = \frac{1}{2} g t^2$

Velocity of the Projectile at any Instant

Suppose that a projectile is fired in a direction making an angle θ with the horizontal by velocity V_i as shown in Fig.

At any instant the velocity of the projectile has two components (1) horizontal component, (2) vertical component. These two components are independent to each other. During the motion of the projectile horizontal component of the velocity remains same, so



$$a_x = 0$$

because we have neglected air resistance and no other force is acting along this direction.

As the projectile moves up under the action of gravity, so

$$a_y = -g$$

As $\quad V_f = V_i + a t$

For two dimensional – motion

$$V_{fx} = V_{ix} + a_x t$$

$\because \quad a_x = 0$

$\therefore \quad V_{fx} = V_{ix} + 0\,(t)$

$V_{fx} = V_{ix} = V_i \cos\theta \quad \text{......... (1)}$

Also, $\quad V_f = V_i + a t$

$V_{fy} = V_{iy} + a_y t$

$V_{iy} = V_i \sin\theta, \; a_y = -g$

$\therefore \quad V_{fy} = V_i \sin\theta - g t \quad \text{......... (2)}$

Velocity of the projectile at any instant is

$$\vec{V} = \vec{V}_f = V_{fx}\hat{i} + V_{fy}\hat{j}$$

Putting the values of V_{fx} and V_{fy}

$$\vec{V} = V_i \cos\theta\, \hat{i} + (V_i \sin\theta - g\,t)\, \hat{j}$$

Its magnitude is

$$V = \sqrt{V_{fx}^2 + V_{fy}^2}$$

or

$$V = \sqrt{(V_i \cos\theta)^2 + (V_i \sin\theta - g\,t)^2}$$

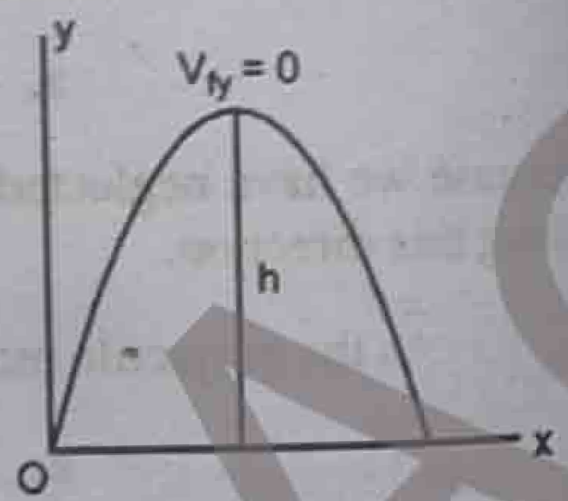
For direction

$$\tan\phi = \frac{V_{fy}}{V_{fx}}$$

$$\phi = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right)$$

Height of the Projectile

The maximum vertical distance which a projectile covers is called height of projectile. In order to determine the maximum height of projectile attains we used the third equation of motion.



Using $2aS = V_f^2 - V_i^2$

or $2a_y y = V_{fy}^2 - V_{iy}^2$ .... (1)

$\because$ $V_{fy} = 0$

As the body comes to rest after reaching highest point

Also $a_y = -g$

$V_{iy} = V_i \sin\theta$

Putting these values in equation (1)

$$\therefore\ 2(-g)y = (0)^2 - (V_i \sin\theta)^2$$

$$-2gy = -V_i^2 \sin^2\theta$$

$$2gy = V_i^2 \sin^2\theta$$

$$\because\ S = y = h$$

$$2gh = V_i^2 \sin^2\theta$$

$$\boxed{\therefore\ h = \frac{V_i^2 \sin^2\theta}{2g}}$$

Time of Flight

"The time taken by the body to cover the distance from the place of its projection to the place where it hits the ground at the same level is called the time of flight."

As the body goes up and comes back to same level, thus covering no vertical displacement.

$\therefore$ $S = y = h = 0$

Also $V_{iy} = V_i \sin\theta$

$a_y = -g$

As $S = V_{iy}\,t + \frac{1}{2} a_y t^2$

$$0 = V_i \sin\theta\, t + \frac{1}{2}(-g)\,t^2$$

$$0 = V_i \sin\theta\, t - \frac{1}{2} g\, t^2$$

$$\frac{1}{2} g\, t^2 = V_i \sin\theta\, t$$

$$\frac{1}{2} g\, t = V_i \sin\theta$$

$$\boxed{t = \frac{2\, V_i \sin\theta}{g}}$$

Time to reach maximum Height

As, $V_f = V_i + a\,t$

$V_{fy} = V_{iy} + a_y\,t$

As, Motion is upward

$\therefore$ $a_y = -g$

$V_{fy} = 0$

$0 = V_i \sin\theta - g\,t$

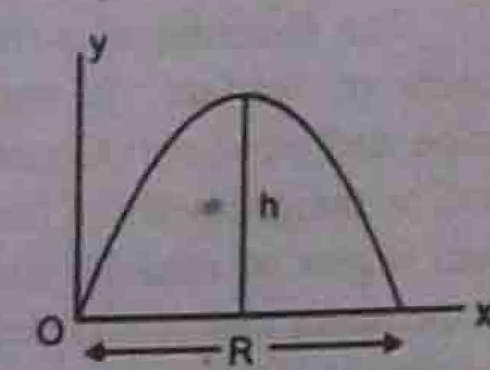
$gt = V_i \sin\theta$

$$t = \frac{V_i \sin\theta}{g}$$

Range of the Projectile

"The maximum distance, which a projectile covers in the horizontal direction is called the range of the projectile. It is denoted by "R".

In order to find "R", we multiply the horizontal component of the velocity with time of flight.



Hence, $R = V_{ix} \times t$

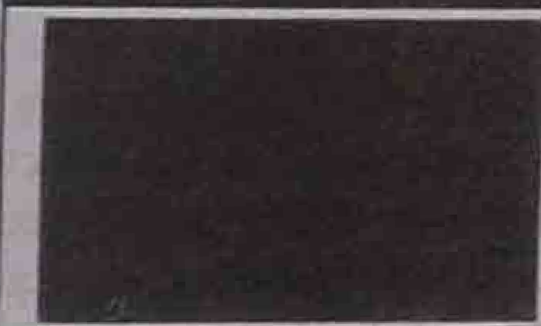
$$= V_i \cos\theta \times \frac{2\, V_i \sin\theta}{g}$$

$$= \frac{V_i^2 (2 \sin\theta \cos\theta)}{g}$$

Interesting Information

A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

Point to Ponder



Water is projected from two rubber pipes at the same speed-from one at an angle of 30° and from the other at 60°. Why are the ranges equal?

Ans. Because the range will be maximum at 45°. But if the angles of projection which exceed or fall short of 45° by equal amounts, the ranges are equal.

$$R = \frac{V_i^2 \sin 2\theta}{g} \qquad (\because\ 2\sin\theta\cos\theta = \sin 2\theta)$$

Thus the range of projectile depends upon the velocity of projection and the angle of projection.

Maximum Range

R will be maximum

when, $\sin 2\theta = 1$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = \frac{90^\circ}{2}$$

$$\theta = 45^\circ$$

$$\therefore \quad R_{max} = \frac{V_i^2}{g}$$

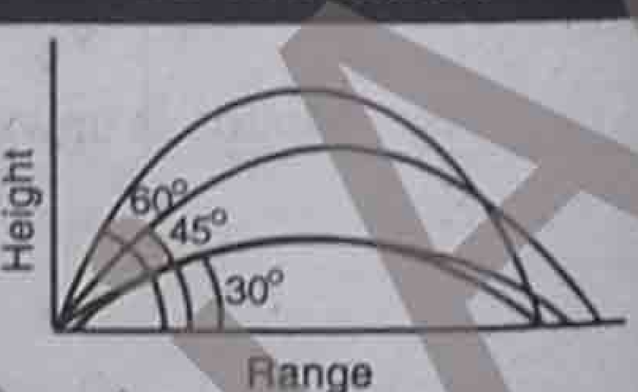
Application to Ballistic Missiles

A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity. An un–powered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory.

As discussed before, a ballistic missile moves in a way that is the result of the superposition of two independent motions: a straight line inertial flight in the direction of the launch and a vertical gravity fall. By law of inertia, an object should sail straight off in the direction thrown, at constant speed equal to its initial speed particularly in empty space. But the downward force of gravity will alter straight path into a curved trajectory. For short ranges and flat Earth approximation, the trajectory is parabolic but the dragless ballistic trajectory for spherical Earth should actually be elliptical. At high speed and for long trajectories the air friction is not negligible and some times the force of air friction is more than gravity. It affects both horizontal as well as vertical motions. Therefore, it is completely unrealistic to neglect the aerodynamic forces.

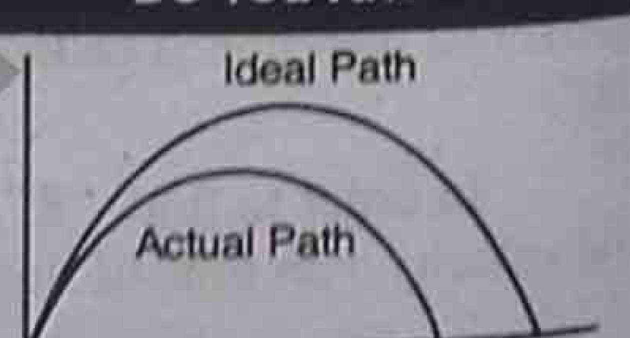
The shooting of a missile on a selected distant spot is a major element of warfare. It undergoes complicated motions due to air friction and wind etc. Consequently the angle of projection cannot be found by the geometry of the situation at the moment of launching. The actual flights of missiles are worked out to high degrees of precision and the result were contained in tabular form. The modified equation of trajectory is too complicated to be discussed here. The ballistic missiles are useful precision, powered and remote control guided missiles are used.

Do You Know?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

Do You Know?



In the presence of air friction the trajectory of a high speed projectile fall short of a parabolic path.

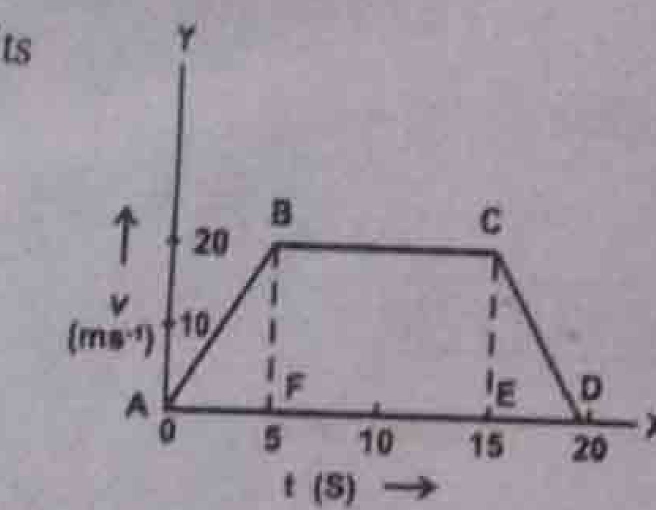
SOLVED EXAMPLES

EXAMPLE 3.1

The velocity time graph of a car moving on a straight road is shown in figure. Describe the motion of the car and find the distance covered.

SOLUTION

The graph shows that car starts from rest i.e., $V_i = 0$ and its velocity becomes 20 m/s in time 5 sec. Then



Average acceleration is given by

$$a = \frac{\Delta V}{\Delta t} = \frac{20}{5} = 4\ m/s^2$$

Motion of car from B to C

The graph shows that car moves with uniform velocity of 20 m/s^2. Here acceleration is zero.

Motion of Car From C to D

The graph shows that the acceleration decreases during last four seconds

and $\quad a = \frac{\Delta V}{\Delta t} = \frac{-20}{4} = -5\ m/s^2$

Negative Sign Shows that Velocity is Decreasing

Distance Covered by Car

Distance covered = Area of trapezium OABCO

Now

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of parallel sides}) \times (\text{distance between parallel sides})$$

$$= \frac{1}{2}(10 + 19) \times 20$$

$$= \frac{1}{2}(29)(20) = 290$$

Thus

Distance covered = 290 m

Note: The distance covered in above example can also be determined by applying.

Distance covered = Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE.

EXAMPLE 3.2

A 1500 kg car has its velocity reduced from 20 ms^{-1} to 15 ms^{-1} in 3.S. How large was the average retarding force.

Data

Mass = m = 1500 kg
Initial velocity = V_i = 20 m s^{-1}
Final velocity = V_f = 15 m s^{-1}
Time taken = t = 3.0 s

To Find

Force = F = ?

SOLUTION

We know that

$$F = \frac{m V_f - m V_i}{t} = \frac{m(V_f - V_i)}{t}$$

Putting values, we get

$$F = \frac{1500(15-20)}{3} = \frac{1500 \times (-5)}{3} = -2500 \text{ N}$$

$$\boxed{F = -2500 \text{ N}}$$

The negative sign indicates the force is retarding.

Result

Force = F = −2500 N
= 2.5 KN

EXAMPLE 3.3

Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 ms^{-1} and 4 ms^{-1} respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 ms^{-1}?

Data

Mass of 1st ball = m_1 = 2 kg
Mass of 2nd ball = m_2 = 3 kg

Velocity of 1st ball = V_1 = 6 m s^{-1}
Velocity of 2nd ball = V_2 = −4 m s^{-1}
Velocity after collision = V_2' = −3 m s^{-1}

To Find

Velocity of 1st ball after collision = V_1' = ?

SOLUTION

Using law of conservation of momentum.

Momentum of the system before collision = Momentum of the system after collision

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

Putting values

$$2(6) + 3(-4) = 2V_1' + 3(-3)$$
$$12 - 12 = 2V_1' - 9$$
$$0 = 2V_1' - 9$$
$$2V_1' = 9$$
$$V_1' = 4.5 \text{ m/s}$$

Result

Velocity of 1st ball after collision = V_1' = 4.5 m/s

EXAMPLE 3.4

A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 ms^{-1} to the right while the second ball is at rest. If the collisions were perfectly elastic what would be the velocity of the two balls after the collision?

Data

Mass of 1st ball = m_1 = 70 g = 0.07 kg
Mass of 2nd ball = m_2 = 140 g = 0.14 kg
Velocity of 1st ball = V_1 = 9 m s^{-1}
Velocity of 2nd ball = V_2 = 0

To Find

Velocity of 1st ball after collision = V_1' = ?
Velocity of 2nd ball after collision = V_2' = ?

SOLUTION

Using $V_1' = \frac{m_1 - m_2}{m_1 + m_2} V_1 + \frac{2 m_2 V_2}{m_1 + m_2}$

$V_1' = \frac{0.07 - 0.14}{0.07 + 0.14} \times 9$

$V_1' = -3 \text{ m/s}$

Now, $V_2' = \frac{2m}{m_1 + m_2} V_1 + \frac{m_2 - m_1}{m_1 + m_2} V_2$

$V_2' = \frac{2 \times 0.14}{0.07 + 0.14} \times 9 + 0$

$V_2' = 6 \text{ ms}^{-1}$

Result

Velocity of 1st ball after collision = $V_1' = -3$ m/s

Velocity of 2nd ball after collision = $V_2' = 6$ m/s

EXAMPLE 3.5

A 100 g golf ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with a 8 kg steel ball, initially at rest. Compute velocities of the ball after collision.

Data

Mass of 1st ball $= m_1 = 100$ g

$= \frac{100}{1000}$ kg

$= 0.1$ kg

Mass of 2nd ball $= m_2 = 8$ kg

Velocity of 1st ball $= V_1 = 20 \text{ m s}^{-1}$

Velocity of 2nd ball $= V_2 = 0$

To Find

Velocity of 1st ball after collision = V_1' = ?

Velocity of 2nd ball after collision = V_2' = ?

SOLUTION

Using $V_1' = \frac{m_1 - m_2}{m_1 + m_2} V_1 + \frac{2m_2 V_2}{m_1 + m_2}$

$V_1' = \frac{0.1 - 8}{0.1 + 8} \times 20 + 0$

$V_1' = -19.5 \text{ m s}^{-1}$

Now, $V_2' = \frac{2m}{m_1 + m_2} V_1 + \frac{m_2 - m_1}{m_1 + m_2} V_2$

$V_2' = \frac{2 \times 0.1}{0.1 + 8} \times 20 + 0$

$V_2' = 0.49 \text{ m s}^{-1}$

$V_2' = 0.5 \text{ m s}^{-1}$

Result

Velocity of 1st ball after collision = $V_1' = -19.5$ m/s

Velocity of 2nd ball after collision = $V_2' = 0.5$ m/s

EXAMPLE 3.6

A hose pipe ejects water at a speed of 0.3 ms^{-1} through a hole of area 50 cm^2. If the water strikes a wall normally, calculate the force on the wall, assuming the velocity of the water normal to the wall is zero after striking.

Data

$V = 0.3 \text{ m s}^{-1}$

$A = 50 \text{ cm}^2$

$= 50 \times 10^{-4} \text{ m}^2$

To Find

Force on the ball $= F = ?$

SOLUTION

Using $F = \frac{m V}{t}$ .... (1)

As $\rho = \frac{m}{V}$

$m = \rho V$

$\therefore \frac{V}{t}$ (volume of water per second) $= AV$

$= 50 \times 10^{-4} \times .3$

$= 0.0015 \text{ m}^3$

Since $\frac{m}{t} = \rho \frac{V}{t}$

$\frac{m}{t} = 1000 \times 0.0015$

$= 1.5$ kg/s

Putting equation (1)

$\therefore \quad F = 1.5\,(.3)$

$= 0.45\ N$

Result

Force on the ball $= F = 0.45\ N$

EXAMPLE 3.7

A ball is thrown with a speed of 30 ms^{-1} in a direction 30° above the horizontal. Determine the height to which it rises, the time of flight and horizontal range.

Data

Angle with horizontal $= \theta = 30^\circ$

Initial speed $= V_i = 30\ m\,s^{-1}$

To Find

Vertical height $= h = ?$

Time of flight $= t = ?$

Horizontal range $= R = ?$

SOLUTION

For Vertical Height

$$h = \frac{V_i^2 \sin^2\theta}{2g}$$

$$h = \frac{(30 \sin 30)^2}{2 \times 9.8}$$

$$= 11.5\ m$$

For Range of Projectile

$$R = \frac{V_i^2 \sin^2\theta}{g}$$

$$R = \frac{(30)^2 \sin 2 \times 30}{9.8}$$

$$R = \frac{(30)^2 \sin 60}{9.8}$$

$$R = \frac{(30)^2 (0.866)}{9.8}$$

$$R = 79.5\ m$$

For Time of Flight

$$t = \frac{2\,V_i \sin\theta}{g}$$

$$t = \frac{2 \times 30 \times \sin 30}{9.8}$$

$$t = 3.1\ sec$$

Result

Vertical height $= h = 11.5\ m$

Horizontal range $= R = 79.5\ m$

Time of flight $= t = 3.1\ sec.$

EXAMPLE 3.8

In example 3.7 calculate the maximum range and the height reached by the ball if the angles of projection are (i) 45° (ii) 60°.

Data

Initial speed $= V_i = 30\ m/s$

Angle with horizontal $= \theta = 30^\circ$

To Find

Maximum range $= R = ?$

Vertical height $= h = ?$

When: (i) $\theta = 45^\circ$

(ii) $\theta = 60^\circ$

SOLUTION

As we know that

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

(i) **When $\theta = 45^\circ$**

$$R = \frac{(30)^2 \sin 2 \times 45^\circ}{9.8}$$

$$= \frac{900 \times \sin 90}{9.8}$$

$$= \frac{900 \times 1}{9.8}$$

$$\boxed{R = 91.8\ m}$$

and vertical height

Using formula

$$h = \frac{V_i^2 \sin^2\theta}{2\,g}$$

$$= \frac{(30)^2 (\sin 45)^2}{2\,(9.8)}$$

$$= \frac{900\,(0.707)^2}{19.6}$$

$$= 23\ m$$

$$\boxed{h = 23\ m}$$

(ii) **When $\theta = 60°$**

We know that

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

$$= \frac{(30)^2 \sin 2 \times 60°}{9.8}$$

$$= \frac{900 \times \sin 120}{9.8}$$

$$= \frac{900 \times 0.866}{9.8}$$

$$\boxed{R = 80\ m}$$

and vertical height is

Using formula

$$h = \frac{V_i^2 \sin^2\theta}{2g}$$

$$= \frac{(30)^2 (\sin 60)^2}{2(9.8)}$$

$$= \frac{900\,(0.866)^2}{19.6}$$

$$= 34.4\ m$$

$$\boxed{h = 34.4\ m}$$

Result

(i) **When $\theta = 45°$**

Maximum range = R = 91.8 m

Vertical height = h = 23 m

(ii) **When $\theta = 60°$**

Maximum range = R = 80 m

Vertical height = h = 34.4 m

SHORT QUESTIONS

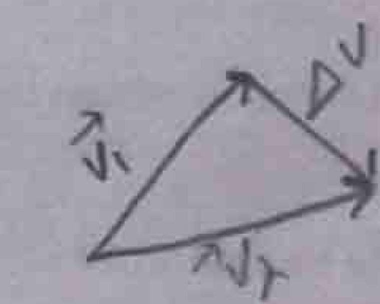
3.1 What is the difference between uniform and variable velocity. From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration.

Ans. **Uniform Velocity:** The velocity of a body is said to be uniform if it covers equal displacement in equal interval of time.

Variable Velocity: The velocity of a body is said to be variable if it covers unequal displacement in unequal interval of time.

Acceleration: From the variable velocity, the rate of change of velocity is called acceleration.

Let a body is moving with velocity $\vec{v}_i$. After small time Δt its velocity changes from $\vec{v}_i$ to $\vec{v}_f$ then the change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. So



$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

SI Unit of Velocity: The SI unit of velocity is m/s or km/hr.

SI Unit of Acceleration: The SI unit of acceleration is m/s^2.

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

Ans. When an object is thrown vertically upward, the sign of acceleration due to gravity is negative relative to velocity. But when the object is thrown downward, the sign of acceleration due to gravity is taken as positive because velocity and acceleration are in same direction.

3.3 Can the velocity of an object reverse direction when acceleration is constant? If so, give an example.

Ans. Yes, the velocity of an object can reverse its direction when acceleration is constant.

Example: When an object is thrown vertically upward then during upward motion its velocity decreases, the direction of velocity will be in upward while direction of acceleration due to gravity will be in downward and when it reach at the highest point its velocity become zero but during downward of object the direction of velocity will be in downward while direction of acceleration due to gravity will again in downward thus we see that in this case the velocity reverse the direction while acceleration is constant.

3.4 Specify the correct statements:

(a) **An object can have a constant velocity even its speed is changing.**

(b) **An object can have a constant speed even its velocity is changing.**

(c) **An object can have a zero velocity even its acceleration is not zero.**

(d) **An object subjected to a constant acceleration can reverse its velocity.**

Ans. (a) It is false statement because an object cannot have a constant velocity even its speed is changing.

(b) It is true when the object is moving along a circular path.

(c) It is true because when an object is thrown vertically upward, at maximum height, velocity is zero but acceleration is not zero, it is $a = g$.

(d) It is true. Yes an object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

Ans. Both the balls have the same speed on striking the ground but time is different. When the velocity of the ball thrown upward with initial velocity v_i, it will has same velocity v_i when it return back and passes the man so as the initial velocities of a ball is same for both cases, therefore the final velocities will also be same.

3.6 Explain the circumstances in which the velocity "v" and acceleration "a" of a car are:

(i) Parallel **(ii) Anti-parallel**

(iii) Perpendicular to one another **(iv) "v" is zero but "a" is not**

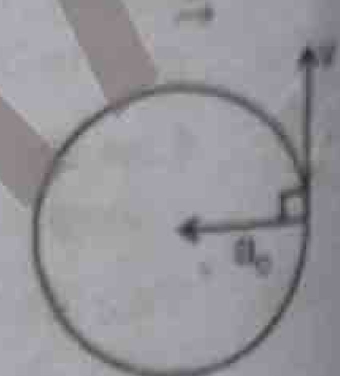
(v) "a" is zero but "v" is not zero

Ans. Following are the circumstances when velocity and acceleration of car:

(i) **Parallel:** When the velocity of a car is increasing along a straight path then velocity and acceleration are parallel to each other.

(ii) **Anti-parallel:** When the velocity of car is decreasing along the straight line then velocity and acceleration are anti-parallel to each other.

(iii) **Perpendicular to one another:** The velocity and acceleration of a car are perpendicular to each other when the car is moving along a circular path.



(iv) **v is zero but a is not zero:** The velocity of a car becomes to zero when the brakes are applied and the car comes to rest due to acceleration in opposite direction.

(v) **a is zero but v is not zero:** Acceleration is zero when the car is moving with uniform acceleration.

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

Ans. Yes, the motion with constant velocity is a special case of motion with constant acceleration. This statement is true.

Explanation: we know that when a body moves with constant velocity then its acceleration will be zero i.e., there is no rate of change of velocity so whenever it moves with constant velocity its acceleration will remain zero that is constant here zero is also a constant quantity. Therefore motion with constant velocity is a special case of motion with constant acceleration.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

Ans. Consider a body of mass "m" moving with velocity v_i. Let a force F is applied on the body which changes the velocity from v_i to v_f then according to 1st equation of motion.

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

But from Newton's second law of motion

$$F = ma$$

$$F = m\left(\frac{v_f - v_i}{t}\right)$$

$$F = \frac{mv_f - mv_i}{t}$$

Where mv_f is the final momentum and mv_i is the initial momentum so,

$$\frac{mv_f - mv_i}{t} = \text{Rate of change of momentum}$$

$$F = \text{Rate of changed momentum}$$

Newton's Second Law of Motion in Terms of Momentum: Newton's second law of motion in terms of momentum states, "the rate of change of momentum is equal to applied force".

3.9 Define impulse and show that how it is related to linear momentum?

Ans. **Impulse:** When a very large force acts on a body for a very short interval of time then the product of such a force and time is called impulse. It is a vector quantity

Impulse $= I = \text{Force} \times \text{Time}$

$$I = F \times \Delta t$$

As we know that

$$F \times \Delta t = mv_f - mv_i$$

So $$I = mv_f - mv_i$$

$$\vec{I} = m\Delta\vec{v}$$

$$\vec{I} = \Delta\vec{P} = \text{Change in momentum}$$

This shows that impulse is equal to change in momentum.

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

Ans. **Law of Conservation of Linear Momentum:** This law states that the total linear momentum of an isolated system remains constant.

Importance of an Isolated System: This law holds good only for isolated system. An isolated system is one at which there is no external force acting. If the system is not isolated but the external forces are very small as compared to interacting forces so this law can also be applied on such a system.

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

Ans. **Elastic Collision:** These collision in which kinetic energy remains constant is called elastic collisions.

Inelastic Collision: These collision in which kinetic energy does not remain constant is called inelastic collisions.

In Case of Bouncing Ball: If the ideal bouncing ball returns to the same height where it is dropped then the collision is elastic collision. If the bouncing ball will not returned to the same height then the collision is inelastic. So due to change of energy, kinetic energy does not remain constant.

For example; when a heavy ball is dropped on to the surface of earth, it rebounds upto very little height because maximum K.E is lost due to friction and also changes into heat and sound energies. So in most cases, the K.E is not conserved. Thus momentum and K.E are conserved in all types of collisions. However the K.E is conserved only in elastic collision.

3.12 Explain what is meant by projectile motion. Derive expressions for

(a) The time of flight (b) The range of projectile.

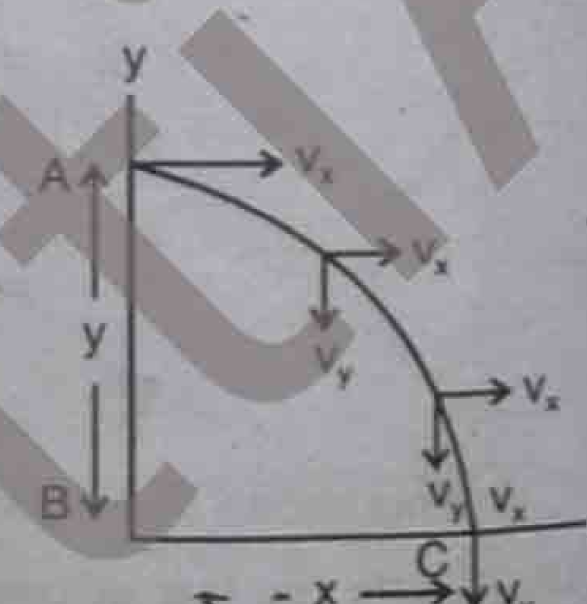
Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

Ans. **Projectile Motion:** When an object is thrown in air making a certain angle with horizontal, so that object moves under the action of gravity and moves along a curved path, is called as "projectile". Its motion is called "projectile motion". Its path is called trajectory. Its path is parabolic. **(OR)** Projectile motion is two dimensional motion under constant acceleration due to gravity.

The body thrown is called projectile and the curved path followed by it is called trajectory.

Examples:

1. Motion of football kicked off by a player.
2. A ball thrown by a cricketer.
3. Missile fired from launching pad.



Consider a body thrown in horizontal direction with horizontal velocity v_x from point A having vertical height 'y'. In the absence of horizontal force, the horizontal components v_x remain constant all along the motion. If the body hits at point 'C' the horizontal distance 'x' covered by the body is given by

$$x = v_x t$$

Where 't' is the time taken by body to move from A to C.

The body not only covers distance in forward direction but also moves down under the action of gravity. The downward vertical velocity of body under the action of gravity goes on increasing continuously. This vertical motion is same as for freely falling body. The distance covered by body in downward direction is AB = y and is given by

$$S = y = v_{iy}t + \frac{1}{2}at^2$$

As the ball at 'A' has only the horizontal velocity so

$$V_{iy} \text{ (initial vertical velocity) } = 0 \quad \text{and} \quad a = g$$

So $$y = \frac{1}{2}gt^2$$

$$\boxed{y = \frac{1}{2}gt^2}$$

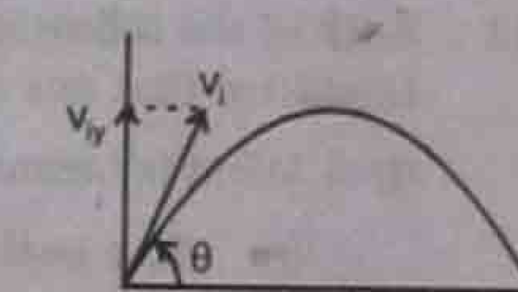
Time of Flight of Projectile: The time taken by body to cover the distance from place of projection to the place where it hits the ground, is called time of flight of projectile. The time of flight can be calculated by using 2nd equation of motion:

$$S = v_i t + \frac{1}{2}gt^2$$

As the ball returns to ground, so net vertical distance is zero. i.e.,

$$S = 0 \quad \text{and} \quad v_i = v_{iy} = v_i \sin\theta$$

The above equation becomes



$$0 = v_i \sin\theta t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = v_i \sin\theta t \quad \text{or} \quad \frac{1}{2}gt = v_i \sin\theta$$

or $$t = \frac{2v_i \sin\theta}{g}$$

Where 't' is the time of flight of projectile.

Range of Projectile: Max. distance which a projectile conversion the horizontal direction is called the range of projectile. In order to find R

$$R = v_{ix} \times t$$

$$= \frac{v_i \cos\theta \times 2v_i \sin\theta}{g}$$

$$= \frac{v_i^2}{g} 2\sin\theta\cos\theta$$

$$R = \frac{v_i^2}{g}\sin 2\theta$$

The formula for the **range of projectile** is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

For maximum horizontal range sin 2θ must have maximum value and we know that the maximum value of sin 2θ is 1.

$$\because \quad \sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

So above equation becomes

$$R_{max.} = \frac{v_i^2 \sin 2(45^\circ)}{g}$$

$$R_{max.} = \frac{v_i^2 \sin 90^\circ}{g}$$

$$R_{max.} = \frac{v_i^2}{g}$$

important

So the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed?

Ans. The speed of the projectile is maximum at the point of projection and also at the point where it hits the ground. While the speed of projectile is minimum when it reaches the maximum height.

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identified that answer.

(i) What is meant by a ballistic trajectory?

(a) The paths followed by an un-powered and unguided projectile is called ballistic trajectory.

(b) The path followed by the powered and unguided projectile is called ballistic trajectory.

(c) The path followed by un-powered but guided projectile.

(d) The path followed by powered and guided projectile.

(ii) What happens when two-body system undergoes elastic collision?

(a) The momentum of the system changes.

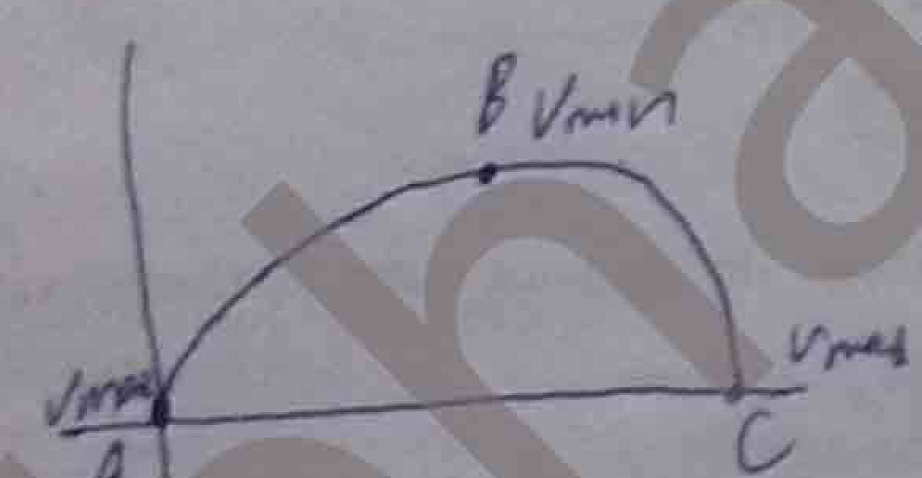
(b) The momentum of the system does not change.

(c) The bodies come to rest after collision.

(d) The energy conservation law is violated.

Ans. (i) (a) is correct.

(ii) (b) is correct.



PROBLEMS WITH SOLUTIONS

important

PROBLEM 3.1 1, 10, 9, 13

A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

METHOD-I

Data

Initial vertical velocity of helicopter $= v_i = 19.6$ m/s

Since initial velocity of the stone is upward and stone moves downward.

∴ Vertical distance travelled by stone $= S = -156.8$ m

To Find

Time taken by stone to reach the ground $= t = ?$

SOLUTION

By using 2[nd] equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$-156.8 = 19.6\,t + \frac{1}{2} \times -9.8\,t^2$$

$$-156.8 = 19.6\,t - 4.9\,t^2$$

$$4.9\,t^2 = -19.6\,t - 156.8 = 0$$

Dividing by 4.9

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t-8)(t+4) = 0$$

$$t - 8 = 0 \quad , \quad t + 4 = 0$$

$$t = 8 \text{ sec.} \quad , \quad t = -4 \text{ sec.}$$

Since time is always positive so ignoring the negative time hence.

Result

Time taken by stone $= t = 8$ sec.

METHOD-II

Data

Initial vertical velocity of helicopter $= v_i = -19.6$ m/s

Vertical distance $= S = 156.8$ m

To Find

Time taken by stone to reach the ground $= t = ?$

SOLUTION

By using the 2nd equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$

$$156.8 = -19.6\,t + \frac{1}{2} \times 9.8\,t^2$$

$$156.8 = -19.6\,t + 4.9\,t^2$$

Divide by 4.9

$$\frac{156.8}{4.9} = \frac{-19.6}{4.9} t + \frac{4.9}{4.9} t^2$$

$$32 = -4t + t^2$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t+4)(t-8) = 0$$

$$t + 4 = 0 \;,\; t - 8 = 0$$

$t = -4$ sec. (neglected), $t = 8$ sec.

Result

Time taken by stone to reach the ground $= t = 8$ sec.

PROBLEM 3.2

Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate

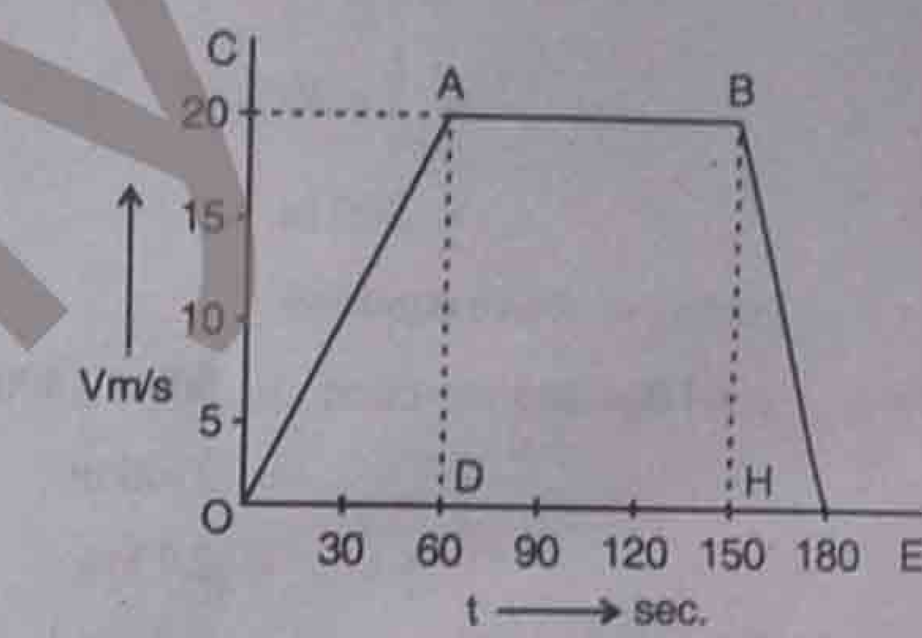
(a) The initial acceleration

(b) The final acceleration

(c) The total distance travelled by the motorcyclist.

SOLUTION

The velocity time graph is as shown.



(a) For initial acceleration

Initial acceleration

$$a_i = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a_i = \frac{\Delta v}{\Delta t}$$

Since $\Delta v = 20$ m/s

$\Delta t = 60$ sec.

So, $a_i = \dfrac{20}{60}$

$= \dfrac{1}{3}$ m/s^2

$a_i = 0.33$ m/s^2

(b) For final acceleration

$$a_f = \frac{\text{Change in velocity}}{\text{Time}}$$

Since $\Delta v = v_f - v_i$

$= 0 - 20$

$= -20$ m/s

and $\Delta t = 30$ sec.

$a_f = \dfrac{-20}{30}$

$a_f = -0.67$ m/s^2

(c) For total distance travelled by motorcyclist

Total distance = Area of ΔOAD + Area of rectangle ABHD + Area of ΔBHE

Thus;

Area of ΔOAD $= \dfrac{1}{2}$ Base $\times$ Height

$= \dfrac{1}{2} \times 60 \times 20$

$= 600$ m

Area of rectangle ABHD = Length $\times$ Breadth

$= 90 \times 20$

$= 1800$ m/s

$$\text{Area of } \Delta BHE = \frac{1}{2}\text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times 30 \times 20$$
$$= 300 \text{ m}$$

Putting in above equation

$$\text{Total distance travelled} = 600 + 1800 + 300$$
$$= 2700 \text{ m}$$
$$= 2.7 \text{ km}$$

Result

(a) Initial acceleration = a_i = 0.33 m/s^2

(b) Final acceleration = a_f = -0.67 m/s^2

(c) Total distance travelled by motorcyclist = 2.7 km

PROBLEM 3.3

A proton moving with speed of $1.0 \times 10^7 \text{ ms}^{-1}$ passes through a 0.02 cm thick sheet of paper and emerges with a speed of $2.0 \times 10^6 \text{ ms}^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Data

Initial speed of proton = v_i = 1.0×10^7 m/s

Distance covered = S = 0.02 cm = 2×10^{-4} m

Final speed of proton = v_f = 2.0×10^6 m/s

To Find

Retardation (negative acceleration) = a = ?

Time taken = t = ?

SOLUTION

For the retardation by using 3rd equation of motion

$$2as = v_f^2 - v_i^2$$
$$2a \times 2 \times 10^{-4} = (2 \times 10^6)^2 - (1.0 \times 10^7)^2$$
$$4 \times 10^{-4} a = 4 \times 10^{12} - 1.0 \times 10^{14}$$
$$= 10^{12}(4 - 1.0 \times 10^2)$$
$$= 10^{12}(4 - 100)$$
$$4 \times 10^{-4} a = -96 \times 10^{12}$$

$$a = \frac{-96 \times 10^{12}}{4 \times 10^{-4}}$$
$$a = -24 \times 10^{16}$$
$$a = -2.4 \times 10^{17} \text{ m/s}^2$$

Negative sign shows retardation.

For the time taken, by using 1st equation of motion

$$v_f = v_i + at$$
$$t = \frac{v_f - v_i}{a}$$
$$= \frac{2.0 \times 10^6 - 1.0 \times 10^7}{-2.4 \times 10^{17}}$$
$$= \frac{10^6(2 - 1.0 \times 10)}{-2.4 \times 10^{17}}$$
$$= \frac{10^6(2 - 10)}{-2.4 \times 10^{17}}$$
$$= \frac{-8 \times 10^6}{-2.4 \times 10^{17}}$$
$$= 3.3 \times 10^{6-17}$$
$$= 3.3 \times 10^{-11} \text{ sec.}$$

Result

Retardation (negative acceleration) = a = $-2.4 \times 10^7 \text{ m/s}^2$

Time taken = t = 3.3×10^{-11} sec.

PROBLEM 3.4

Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the magnitude of ratio of their velocities after the spring has been released?

Data

1st mass = m_1

2nd mass = m_2

Initial velocity of mass m_1 = v_i = 0

Initial velocity of mass m_2 = v_i = 0

To Find

Ratio of their velocities = $\frac{v_1}{v_2}$ = ?

SOLUTION

According to law of conservation momentum

$$m_1v_i + m_2v_i = m_1v_1 + m_2v_2$$

Therefore;

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

Result

Hence after releasing the spring, the ratio of magnitude of their velocities is equal to the inverse ratio of their masses.

PROBLEM 3.5

An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of 1.0×10^{-4} ms^{-1} and at a rate of 1.0×10^{-13} kgs^{-1}. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

(a) If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

(b) If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

Data

Mass of amoeba $= m = 1.0 \times 10^{-12}$ kg

Speed of ejected water $= v = 1.0 \times 10^{-4}$ m/s

Rate of water $= \frac{m}{t} = 1.0 \times 10^{-13}$ kg/s

To Find

(a) Acceleration of amoeba $= a = ?$

(b) Force of surrounding water $= F = ?$

SOLUTION

(a) By formula

$$\boxed{F = \frac{m}{t} \times v}$$

$$F = 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17}\ N$$

So by second law of motion

$$F = ma$$

$$a = \frac{F}{m}$$

$$a = \frac{1.0 \times 10^{-17}}{1.0 \times 10^{-12}}$$

$$= 1.0 \times 10^{-17+12}$$

$$= 1.0 \times 10^{-5}\ m/s^2$$

(b) The force of surrounding water is

$$F = \frac{m}{t} \times v$$

$$= 1.0 \times 10^{-13} \times 1.0 \times 10^{-4}$$

$$= 1.0 \times 10^{-17}\ N$$

Result

(a) Acceleration of amoeba $= a = 1.0 \times 10^{-5}$ m/s^2

(b) Force of surrounding water $= F = 1.0 \times 10^{-17}$ N

PROBLEM 3.6

A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3 ms^{-1}, how fast will the can be going?

Data

Mass of can $= m_1 = 40$ g $= 0.04$ kg

Mass of wooden block $= m_2 = 200$ g $= 0.2$ kg

Speed of wooden block $= v_2' = 3$ m/s

To Find

Speed of can $= v_1' = ?$

SOLUTION

According to law of conservation of momentum

Momentum before explosion = Momentum after explosion

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$0 = m_1v_1' + m_2v_2'$$

$$m_1v_1' = -m_2v_2'$$

$$\boxed{v_1' = -\frac{m_2v_2}{m_1}}$$

$$= -\frac{0.2\times 3}{0.04}$$

$$= -15\ m/s$$

Result

Speed of can $= v_1' = 15\ m/s$

–ve sign shows that the can and wooden block moves in opposite direction.

PROBLEM 3.7

An electron ($m = 9.1\times 10^{-31}$ kg) traveling at $2.0\times 10^{7}\ ms^{-1}$ undergoes a head on collision with a hydrogen atom ($m = 1.67\times 10^{-27}$ kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom?

Data

Mass of electron $= m_1 = 9.1\times 10^{-31}$ kg

Velocity of electron $= v_1 = 2.0\times 10^{7}$ m/s

Mass of hydrogen atom $= m_2 = 1.67\times 10^{-27}$ kg

Velocity of hydrogen atom $= v_2 = 0$

To Find

Velocity of hydrogen atom $= v_2' = ?$

SOLUTION

$$\boxed{v_2' = \frac{2m_1v_1}{m_1+m_2}}$$

$$v_2' = \frac{2\times 9.1\times 10^{-31}\times 2.0\times 10^{7}}{9.1\times 10^{-31}+1.67\times 10^{-27}}$$

$$= \frac{36.4\times 10^{-31+7}}{10^{-27}(9.1\times 10^{-4}+1.67)}$$

$$= \frac{36.4\times 10^{-24+27}}{0.00091+1.67}$$

$$= \frac{36.4\times 10^{3}}{1.67091}$$

$$= 21.78\times 10^{3}$$

$$= 2.18\times 10^{4}\ m/s$$

Result

Velocity of hydrogen atom $= v_2' = 2.18\times 10^{4}$ m/s

PROBLEM 3.8

A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with a stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

Data

Mass of truck $= m_1 = 2500$ kg

Velocity of truck $= v_1 = 21$ m/s

Mass of car $= m_2 = 1000$ kg

Velocity of car $= v_2 = 0$

To Find

Common velocity after collision $= v = ?$

SOLUTION

According to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Since $v_1' = v_2' = v$

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$m_1v_1 + m_2v_2 = (m_1+m_2)v$$

$$\boxed{v = \frac{m_1v_1+m_2v_2}{m_1+m_2}}$$

Putting values

$$v = \frac{2500\times 21 + 1000\times 0}{2500+1000}$$

$$= \frac{52500}{3500}$$

$$v = 15\ m/s$$

Result

Common velocity of truck and car after collision $= v = 15$ m/s

PROBLEM 3.9

Two blocks of masses 2.0 kg and 0.5 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring in 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

Data

Mass of 1st block $= m_1 = 2.0$ kg

Mass of 2nd block $= m_2 = 0.5$ kg

Elastic potential energy $= P.E = 10$ J

To Find

Velocity of mass m_1 $= v'_1 = ?$

Velocity of mass m_2 $= v'_2 = ?$

SOLUTION

According to law of conservation of energy

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$\frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = 10$$

$$m_1v_1'^2 + m_2v_2'^2 = 20$$

$$2v_1'^2 + 0.5v_2'^2 = 20 \quad \text{...... (i)}$$

And according to law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

$$m_1v'_1 + m_2v'_2 = 0$$

$$2v'_1 + 0.5v'_2 = 0$$

$$0.5v'_2 = -2v'_1$$

$$v'_2 = -\frac{2v'_1}{0.5} = -4v'_1$$

$$\boxed{v'_2 = -4v'_1}$$

Putting eq. (i)

$$2v_1'^2 + 0.5(-4v'_1)^2 = 20$$

$$2v_1'^2 + 0.5(16v_1'^2) = 20$$

$$2v_1'^2 + 8v_1'^2 = 20$$

$$10v_1'^2 = 20$$

$$v_1'^2 = \frac{20}{10}$$

$$v'_1 = 1.41 \text{ m/s}$$

and $v'_2 = -4v_1$

$= -4(1.41)$

$v'_2 = -5.65$ m/s

Result

Velocity of mass m_1 $= v'_1 = 1.41$ m/s

Velocity of mass m_2 $= v'_2 = -5.65$ m/s

PROBLEM 3.10

A football is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Data

Angle with horizontal $= \theta = 30^\circ$

Horizontal distance $= R = 40$ m

The value of g $= 9.8 \text{ m/s}^2$

To Find

Initial speed of ball $= v_i = ?$

SOLUTION

By formula

$$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$$

$$v_i^2 = \frac{R \times g}{\sin 2\theta}$$

$$= \frac{40 \times 9.8}{\sin 2(30)}$$

$$= \frac{392}{0.866}$$

$$\sqrt{v_i^2} = \sqrt{452.64}$$

$$v_i = 21.27$$

$$= 21.3 \text{ m/s}$$

Result

Initial velocity of ball $= v_i = 21.3$ m/s

PROBLEM 3.11

A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

Data

Initial horizontal velocity $= v_{ix} = 21$ m/s

Initial vertical velocity $= v_{iy} = 0$

Vertical distance $= y = 10$ m

To Find

Horizontal distance $= R = x = ?$

Velocity to hit the ground $= v = ?$

SOLUTION

Formula for horizontal distance

$$\boxed{x = v_{ix} \times t} \quad \text{...... (i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2$$

$$10 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$$10 = 4.9\, t^2$$

$$t^2 = \frac{10}{4.9}$$

$$t^2 = 2.04$$

$$t = 1.42 \text{ sec.}$$

Therefore; putting in eq. (i)

$$x = R = v_{ix} \times t$$

$$= 21 \times 1.42$$

$$= 29.8$$

$$= 30 \text{ m}$$

For velocity

$$v = \sqrt{v_{fx}^2 + v_{fy}^2}$$

As $v_{fx} = v_{ix} = 21$ m/s

and $v_{fy} = v_{iy} + gt$

$$= 0 + 9.8 \times 1.42$$

$$v_{fy} = 13.91 \text{ m/s}$$

So $v = \sqrt{(21)^2 + (13.91)^2}$

$$= \sqrt{441 + 193.48}$$

$$= \sqrt{634.48}$$

$$v = 25.1 \text{ m/s}$$

Result

Horizontal distance $= x = R = 30$ m

Velocity to hit the ground $= v = 25$ m/s

PROBLEM 3.12

A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

(b) How long was it in air?

Data

Height of bomber $= y = 490$ m

Horizontal velocity $= v_{ix} = 300$ km/hr

$$= \frac{300 \times 1000}{3600}$$

$$= 83.3 \text{ m/s}$$

To Find

(a) Horizontal distance $= x = R = ?$

(b) Time in air $= t = ?$

SOLUTION

(a) For horizontal distance

$$\boxed{x = v_{ix} \times t} \quad \text{...... (i)}$$

For time

$$y = v_{iy}t + \frac{1}{2}gt^2 \qquad \text{Since } v_{iy} = 0$$

$$490 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$$490 = 4.9\, t^2$$

$$t^2 = \frac{490}{4.9}$$

$$t^2 = 100$$

$$t = 10 \text{ sec.}$$

So putting in eq. (i), we get

$$x = v_{ix} \times t$$

$$x = 10 \times 83.3$$

$$= 833 \text{ m}$$

(b) Time in air $= t = 10$ sec.

Result

(a) Horizontal distance $= x = 833$ m

(b) Time in air $= t = 10$ sec.

PROBLEM 3.13

Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

Data

The given that

Horizontal range = Maximym height

To Find

Angle of projection $= \theta = ?$

SOLUTION

As we know that the range of projection is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

and maximum height is

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

Therefore; $R = h$

$$\frac{v_i^2 \sin 2\theta}{g} = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2 \theta}{2}$$

Since $\sin 2\theta = 2 \sin\theta \cos\theta$

$$2 \sin\theta \cos\theta = \frac{\sin^2 \theta}{2}$$

$$2 \cos\theta = \frac{\sin\theta}{2}$$

$$2 \times 2 = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

$$\theta = 75.9$$

$$= 76°$$

Result

Angle of projection $= \theta = 76°$

PROBLEM 3.14

Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

SOLUTION

Let the two angles 30° and 60° which exceed or fall short of 45° by equal of 15°.

Now we have to find, the ranges at these two angles so the range of projectile is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

When $\theta = 30°$

$$R_1 = \frac{v_i^2 \sin 2(30°)}{g}$$

$$R_1 = \frac{v_i^2 \sin 60°}{g}$$

$$R_1 = \frac{0.866\, v_i^2}{g}$$

And when $\theta = 60°$

$$R_2 = \frac{v_i^2 \sin 2(60°)}{g}$$

$$R_2 = \frac{v_i^2 \times 0.866}{g}$$

$$R_2 = \frac{0.866\, v_i^2}{g}$$

Thus $R_1 = R_2$

Result

Hence for angle of projection which exceed or fall short of 45° by equal amounts, the ranges are equal.

PROBLEM 3.15

A SLBM (submarine launched ballistic missile) is fired from a distance of 3000 km. If the Earth were flat and the angle of launch is 45° with horizontal, find the time taken by SLBM to hit the target and the velocity with which the missile is fired.

Data

Horizontal distance $= x = 3000$ km

$= 3000 \times 1000$

$= 3 \times 10^6$ m

Angle of launch $= \theta = 45°$

To Find

Time taken by SLBM to hit the ground $= t = ?$

Velocity of the missile $= v_i = ?$

SOLUTION

For velocity of the missile

$$R = x = \frac{v_i^2 \sin 2\theta}{g}$$

$$\boxed{v_i^2 = \frac{x \times g}{\sin 2\theta}}$$

$$v_i^2 = \frac{3 \times 10^6 \times 9.8}{\sin 2(45^\circ)}$$

$$v_i^2 = \frac{29.4 \times 10^6}{\sin 90^\circ} \qquad \text{Since} \quad \sin 90^\circ = 1$$

$$v_i^2 = 29.4 \times 10^6$$

$$v_i = 5.42 \times 10^3 \text{ m/s}$$

$$= 5.42 \text{ km/s}$$

For time

$$\boxed{t = \frac{2v_i \sin\theta}{g}}$$

$$t = \frac{2(5.42 \times 10^3) \sin 45^\circ}{9.8}$$

$$= \frac{10.84 \times 10^3 \times 0.707}{9.8}$$

$$t = 0.982 \times 10^3$$

$$t = 782 \text{ sec.}$$

$$t = 13 \text{ min.}$$

Result

Time taken by SLMB to hit the ground $= t = 13$ min.

Velocity of the missile $= v_i = 5.42$ km/s

Chapter 4

WORK AND ENERGY

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

★ Understand the concept of work in terms of the product of a force and displacement in the direction of the force.
★ Understand and derive the formula Work = wd = mgh for work done in a gravitational field near Earth's surface.
★ Relate power to work done.
★ Define power as the product of force and velocity.
★ Quote examples of power from everyday life.
★ Explain the two types of mechanical energy.
★ Understand the work-energy principle.
★ Derive an expression for absolute potential energy.
★ Define escape velocity.
★ Give examples of conservation of energies from everyday life.
★ Describe some non-conventional sources of energy.

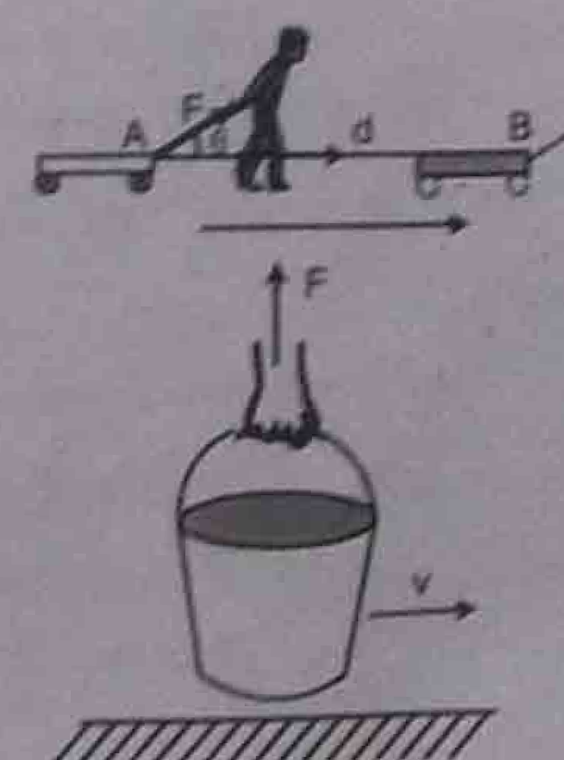
Q.1 Define the term work done. Describe the special cases when the work done is positive, negative and zero.

Ans. WORK DONE BY A CONSTANT FORCE

Work done on a body by a constant force is defined as;

"The product of the magnitude of the displacement and the component of the force in the direction of the displacement."

Consider, an object which is being pulled by a constant force F, at an angle θ to the direction of motion. The force moves the object from A to B through a displacement $\vec{d}$, as shown in figure.



(a)

Since $\quad W = Fd$

$$W = (F\cos\theta)\, d$$

$$W = F d \cos\theta$$

Where $F\cos\theta$ is the component of the force in the direction of $\vec{d}$. So,

$$W = \vec{F} \cdot \vec{d}$$