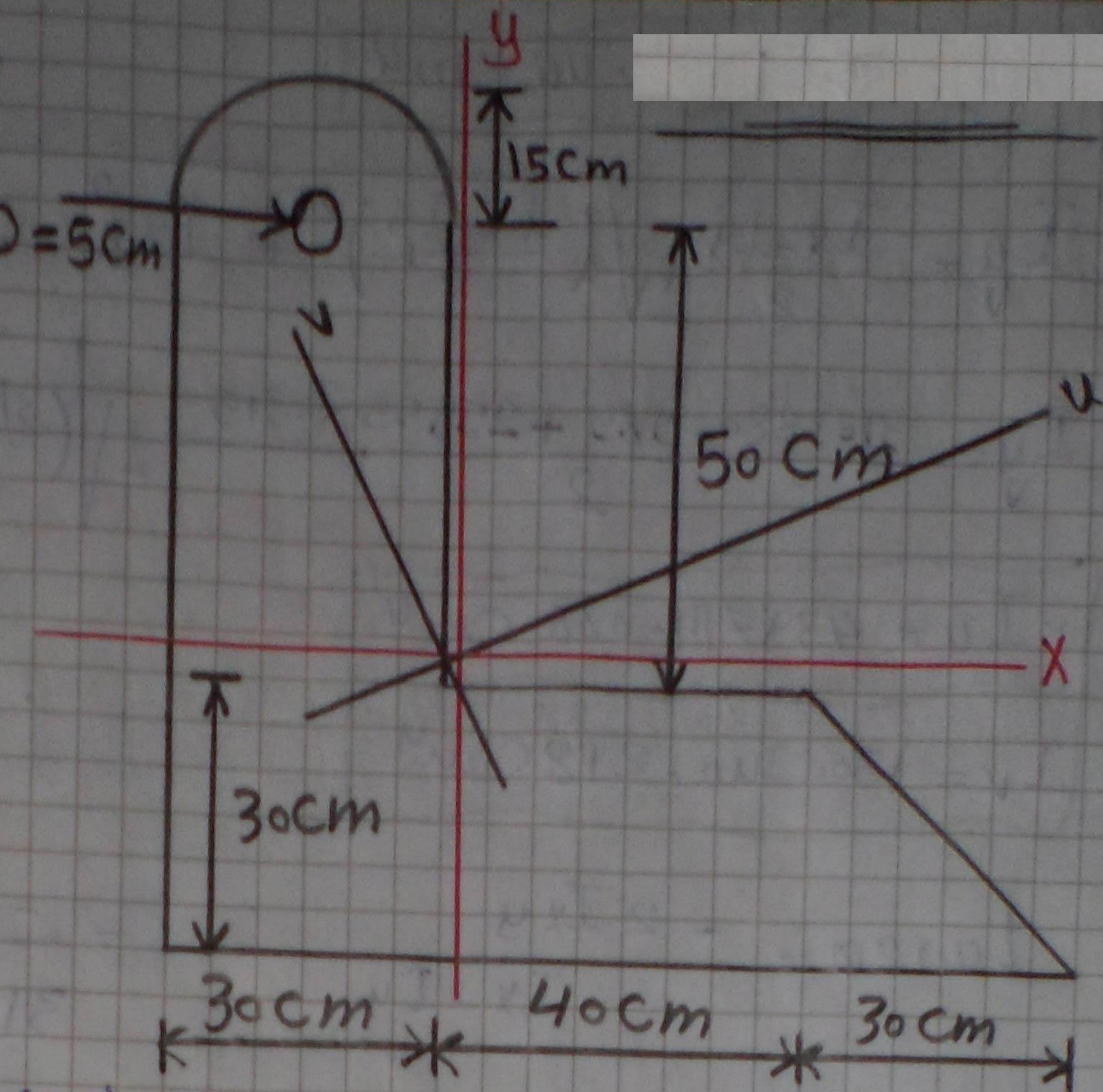
ex1:

السكشن الثالث

Find $I_x, I_y, I_u, I_v, I_{xy}$



$$\bar{X} = \frac{\Sigma A \bar{X}}{\Sigma A}$$

$$\bar{y} = \frac{\Sigma A \bar{y}}{\Sigma A}$$

$$I_x = \left[I_{x_1} + A\bar{y}^2\right]$$

$$I_y = \left[I_{y_1} + A\bar{X}^2\right]$$

$$I_{xy} = \left[I_{x_1y_1} + A_1 \bar{x}_1 \bar{y}_1\right]$$

$$\bar{X} = \frac{353.429(15) + 1500(15) + 2100(35) + 450(80) - 19.635(15)}{4383.79}$$

$$\boxed{\therefore \bar{X} = 31.25\ \text{Cm}}$$

$$\bar{y} = \frac{353.429\left(\frac{4\times15}{3\pi} + 80\right) + 1500(55) + 2100(15) + 450(10) - 19.635(80)}{4383.79}$$

$$\boxed{\therefore \bar{y} = 33.636\ \text{Cm}}$$

$$I_x = \left[0.11(15^4) + 353.429(86.366-33.636)^2\right] + \left[\frac{30\times50^3}{12} + 1500(55-33.636)^2\right]$$
$$+\left[\frac{70(30^3)}{12} + 2100(15-33.636)^2\right] + \left[\frac{30\times30^3}{36} + 450(10-33.636)^2\right] - \left[\frac{\pi(5^4)}{64} + 19.635(80-33.636)^2\right]$$

$$\boxed{\therefore I_x = 3103880.969\ \text{Cm}^4}$$

$$I_y = \left[\frac{\pi(15^4)}{8} + 353.429(15-31.25)^2\right] + \left[\frac{50\times30^3}{12} + 1500(15-31.25)^2\right]$$
$$+\left[\frac{30\times70^3}{12} + 2100(35-31.25)^2\right] + \left[\frac{30^4}{36} + 450(80-31.25)^2\right] - \left[\frac{\pi(5^4)}{4} + 19.635(15-31.25)^2\right]$$

$$\boxed{\therefore I_y = 2595570.315\ \text{Cm}^4}$$

$$I_{xy} = \left[0 + 353.429(15-31.25)(86.366-33.636)\right] + \left[0 + 1500(15-31.25)(55-33.636)\right]$$
$$+\left[0 + 2100(35-31.25)(15-33.636)\right] +$$
$$+\left[\frac{30^4}{72} + 450(80-31.25)(10-33.636)\right] - \left[0 + 19.635(15-31.25)(80-33.636)\right]$$

$$\boxed{I_{xy} = -1526370.44\ cm^4}$$

$$I_{\substack{u\\v}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{\substack{u\\v}} = \frac{3103880.969 + 2595570.315}{2} \pm \sqrt{\left(\frac{3103880.969 - 2595570.315}{2}\right)^2 + (1526370.44)^2}$$

$$\boxed{\therefore I_u = 4397111.066\ cm^4}$$

$$\boxed{I_v = 1302340.218\ cm^4}$$

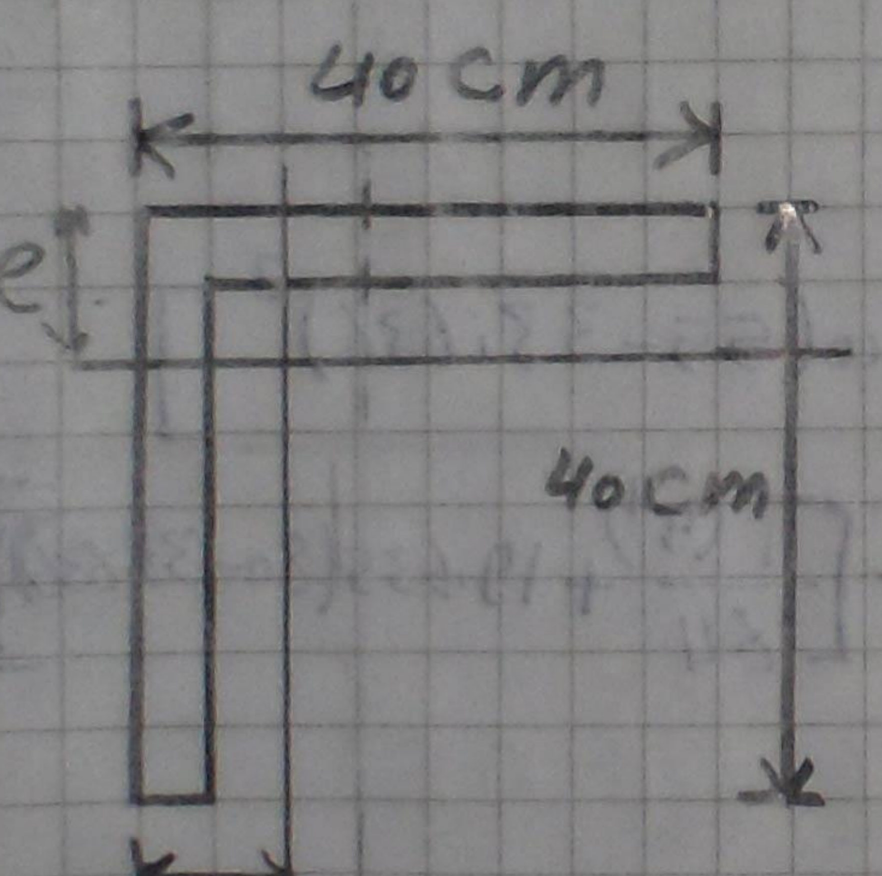
$$\tan 2\theta = \frac{-2 I_{xy}}{I_x - I_y} = \frac{-2(-1526370.44)}{3103880.969 - 2595570.315}$$

$$\tan 2\theta = \frac{+}{+}\ 6.006$$

$$2\theta = 80.546^\circ$$

$$\boxed{\therefore \theta = 40.27^\circ}$$

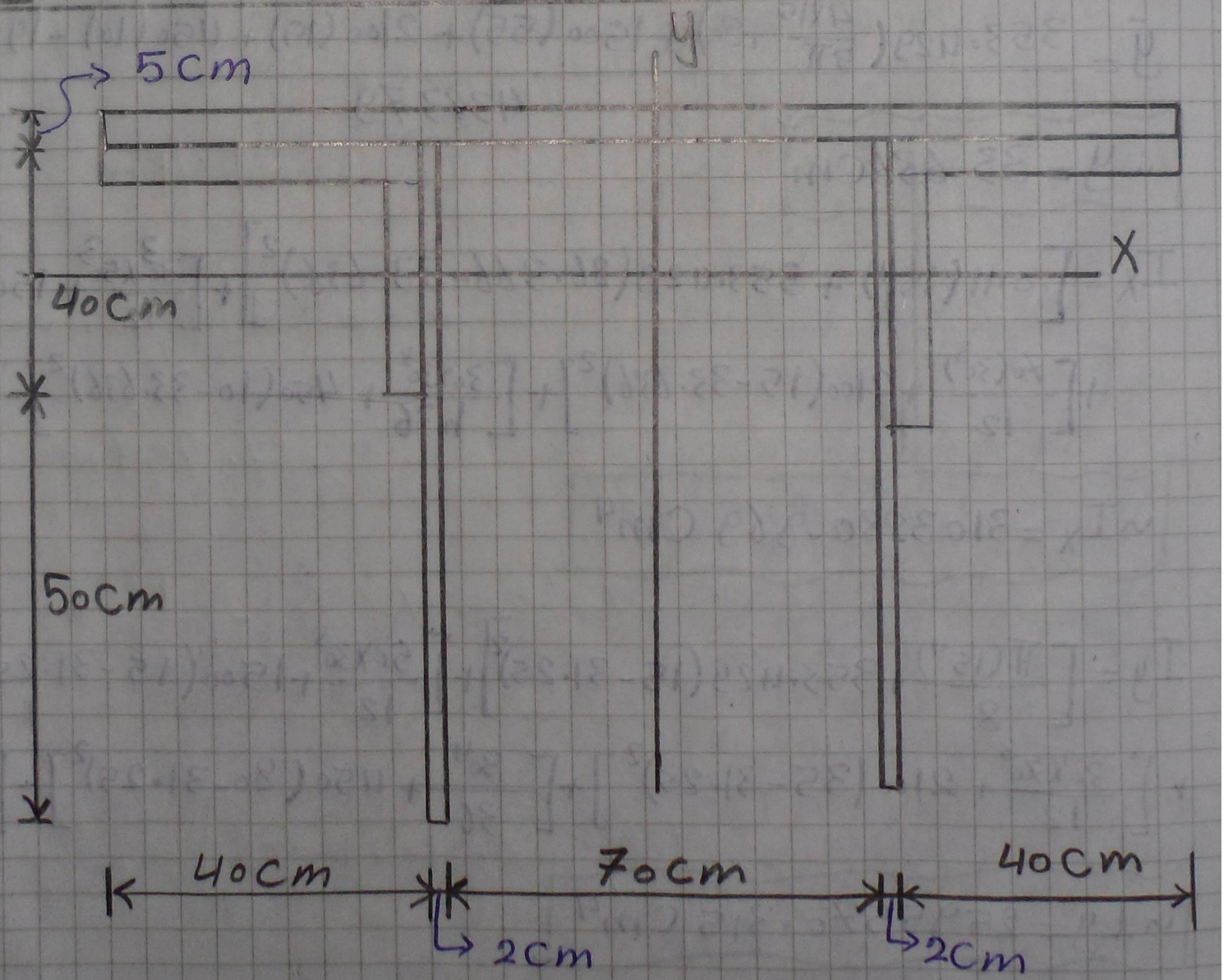
ex:



$I_u = 8000\ cm^4$

$I_v = 2000\ cm^4$

$e = 15\ cm$



$$\boxed{\bar{X} = 77\,cm}$$

$$\bar{y} = \frac{\Sigma A\acute{y}}{\Sigma A} = \frac{(154\times5)(92.5) + 2(2\times90)(45) + 2(5\times40)(87.5) + 2(35\times5)(67.5)}{770 + 2\times180 + 2\times200 + 175\times2}$$

$$\boxed{\therefore \bar{y} = 77.686\,cm}$$

* الفكرة ع المسألة دي هواه احنا معنا ال I_x, I_y لحرف ال (L) بالطريقه العادية هنجيبها كالأتى :

$$I_u = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \rightarrow (1)$$

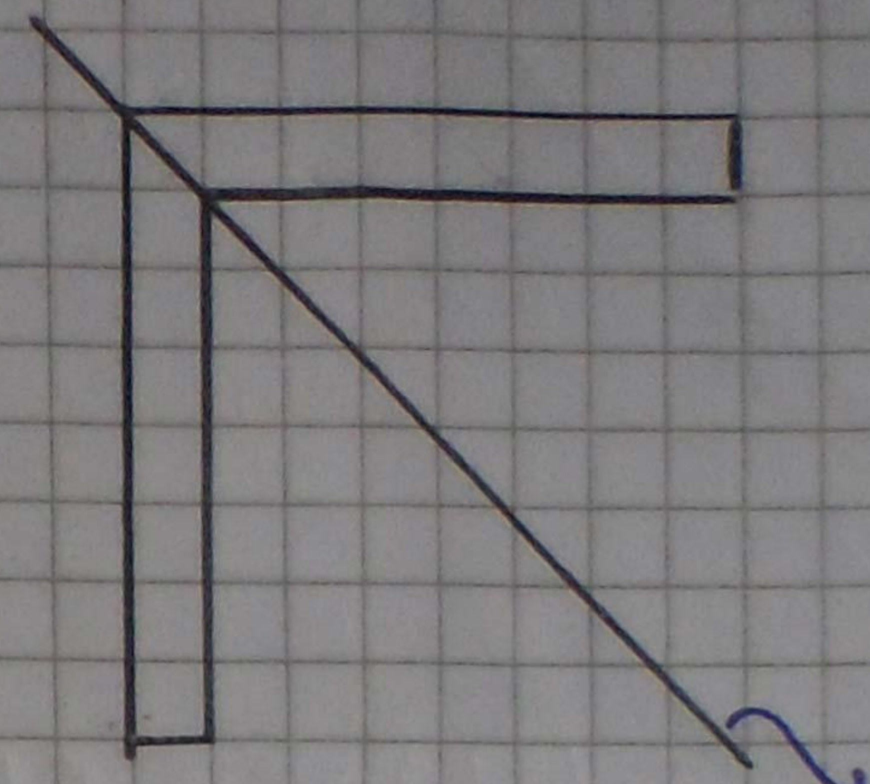
$$I_v = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \rightarrow (2)$$

(1) + (2)

$$I_u + I_v = I_x + I_y = 2 I_x$$

$$I_x = \frac{I_u + I_v}{2} = \frac{8000 + 2000}{2}$$

$$\boxed{I_x = I_y = 5000\,cm^4} \rightarrow$$ لحرف ال (L)



الشكل متماثل حول
هذا المحور
ونفس الابعاد فالجزئية يبقى ال I_x, I_y
متساويه

$$I_x = (I_{x_1} + A\bar{y}^2)$$

$$I_x = \left[\frac{145\times5^3}{12} + 770(92.5 - 77.686)^2\right] + 2\left[\frac{40\times5^3}{12} + 200(87.5 - 77.686)^2\right]$$
$$+ 2\left[\frac{5\times35^3}{12} + 175(67.5 - 77.686)^2\right] + 2\left[\frac{2\times90^3}{12} + 180(45 - 77.686)^2\right]$$

$$\boxed{\therefore I_x = 90507.757\,cm^4}$$

$$I_y = [I_{y_1} + A\bar{x}^2]$$

$$I_y = \left[\frac{5\times145^3}{12} + 770(0)\right] + \left[\frac{5\times40^3}{12} + 200(57 - 77)^2\right]2$$
$$+ 2\left[\frac{35\times5^3}{12} + 175(39.5 - 77)^2\right] + 2\left[\frac{90\times2^3}{12} + 180(36 - 77)^2\right]$$

$$\boxed{\therefore I_y = 2582560.417\,cm^4}$$

$$\boxed{I_u = I_y} \qquad \boxed{I_v = I_x}$$

$$\boxed{I_{xy} = 0}$$